

# Good Idiosyncratic Volatility, Bad Idiosyncratic Volatility, and the Cross-Section of Stock Returns

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July, 2023

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# Good Idiosyncratic Volatility, Bad Idiosyncratic Volatility, and the Cross-Section of Stock Returns

## Abstract

We decompose the idiosyncratic volatility of stock returns into “good” and “bad” volatility components, which are associated with positive and negative returns, and estimate a cross-sectional model for expected good minus bad volatility using firm characteristics. Compared to expected idiosyncratic skewness, expected good minus bad volatility not only more accurately measures conditional idiosyncratic skewness, but also yields stronger return predictability. Importantly, the negative relationship between expected good minus bad volatility and future stock returns remains significant when controlling for expected idiosyncratic skewness and exposure to skewness-related factors. Furthermore, the distinction between good and bad volatility provides new evidence on the role that growth options play behind the strong negative relationship between expected good minus bad volatility and stock returns. In particular, our results suggest that growth options earn lower returns mainly because they predict positive idiosyncratic skewness, which is attractive to investors. Although investors may dislike extreme losses more than gains, we do not find it critical to our results.

Keywords: idiosyncratic skewness, good volatility, bad volatility, cross-sectional stock returns, risk factors, growth options

JEL Code: G10, G12, G13, G17, G31

# 1 Introduction

Finance research has emphasized the importance of idiosyncratic skewness for cross-sectional stock returns. Stocks with positive firm-specific skewness occasionally pay large returns. They may be appealing to investors and hence earn low average returns (e.g., Barberis and Huang, 2008; Boyer et al., 2010; Conrad et al., 2013 and Amaya et al., 2015). Although the empirical relationship between idiosyncratic skewness and expected stock returns has been established, measuring conditional idiosyncratic skewness is still challenging because higher moments are sensitive to outliers. Recent research (e.g, Barinov, 2018 and Langlois, 2020) also suggest that the relationship between idiosyncratic skewness and stock returns might be explained by exposure to skewness-related factors. Moreover, notwithstanding a number of research on the empirical relationship between idiosyncratic skewness and stock returns, there has been less thorough research on distinguishing economic mechanisms that make stocks with positively skewed returns attractive to investors.

Setting against this background, this paper develops a more accurate measure of conditional return asymmetry, which not only yields stronger return predictability for cross-sectional stock returns, but also provides insights of economic mechanisms that underlie the negative relationship between idiosyncratic skewness and stock returns. Our methodology builds on estimators of idiosyncratic return volatility from realizations of positive and negative price increments (e.g., Barndorff-Nielsen and Shephard, 2002; Barndorff-Nielsen et al., 2010 and Patton and Sheppard, 2015). “Good” volatility is the volatility that is associated with positive asset returns, while “bad” volatility is the volatility that is associated with negative asset returns. The difference between good and bad volatility, i.e., good minus bad volatility, could also be used to provide estimates for conditional return asymmetry and to investigate relevant economic mechanisms.

We mainly contribute to the literature in three aspects. First, we propose expected good minus bad volatility as a better measure of the conditional asymmetry in returns than expected idiosyncratic skewness. A major step in establishing the empirical relationship between idiosyncratic skewness and stock returns is to construct measures of conditional idiosyncratic skewness<sup>1</sup>. The

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<sup>1</sup>Because idiosyncratic skewness is strongly time-varying (see Harvey and Siddique, 1999), lagged realized measures alone cannot well capture conditional return asymmetry. In the appendix, we also report results based on lagged realized skewness.

third moments of asset returns may be susceptible to abnormal returns. Feunou et al. (2016) and Bollerslev et al. (2020) suggest that realized good minus bad volatility may be a valid measure of realized skewness. However, existing studies haven't examined the relevance of using good minus bad volatility to measure physical conditional idiosyncratic skewness at the individual stock level. To capture ex-ante variation in physical good minus bad volatility, we run cross-sectional regressions of good minus bad volatility on lagged good minus bad volatility and a set of firm characteristics<sup>2</sup>. Compared with other approaches in forecasting realized good minus bad volatility and realized skewness, we offer the novel finding that expected good minus bad volatility could be a better predictor for not only realized good minus volatility but also realized idiosyncratic skewness.

Second, the expected good minus bad volatility measure has a strong negative relationship with stock returns in both portfolio sorts and Fama-MacBeth regressions. Sorting individual stocks into portfolios based on expected good minus bad volatility yields larger differences in subsequent portfolio returns than sorting by expected idiosyncratic skewness measure of Boyer et al. (2010). Moreover, such differences cannot be explained by expected idiosyncratic skewness and exposure to risk factors. Recent papers (Barinov, 2018; Langlois, 2020) propose the aggregate volatility (FVIX) factor and predicted skewness factor (PSS), which may explain why stocks with positive idiosyncratic skewness earn low returns. When controlling for exposure to these factors, the return predictability of expected idiosyncratic skewness is insignificant. Nonetheless, the return predictability of expected good minus bad volatility remains significant. Since expected good minus bad volatility yields better forecasting performance and stronger asset pricing results, our paper highlights the benefits of using expected good minus bad volatility to measure conditional skewness in individual stock returns. The use of expected good minus bad volatility may lead to more significant portfolio returns and more effective risk hedging for real-world investors whose investment strategies are exposed to asymmetry in stock returns.

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<sup>2</sup>Huang and Li (2019) find a positive relationship between individual stocks' implied variance asymmetry, defined as the difference between upside and downside risk-neutral semivariances extracted from out-of-the-money options, and future stock returns. In contrast, we discover a strong negative relationship between conditional idiosyncratic skewness and stock returns. Option-implied risk-neutral skewness is affected by investors' risk preferences and only available for a subsample of firms with sufficient option data, which may make option-implied measures subject to more liquidity and limits to arbitrage issues. Our conditional idiosyncratic skewness measure covers a broader sample of firms and may capture information different from risk-neutral skewness. Thus, the relevant economic mechanisms to explain our results also differ.

Third, the distinction between good and bad volatility also facilitates disentangling mechanisms behind the negative relationship between idiosyncratic skewness and stock returns. Trigeorgis and Lambertides (2014), Del Viva et al. (2017), and Bali et al. (2020) have argued that the flexibility

2008). Errors in the probability weighting of investors cause them to overvalue stocks that have a small probability of a large positive return. Our results are also consistent with the optimal belief framework of Brunnermeier and Parker (2005) and the context of a market with underdiversified investors, who have preferences for skewed returns (Mitton and Vorkink, 2007).

Our paper relates to a growing literature that studies the implications of good and bad volatility for asset valuations. For example, Feunou et al. (2018) and Feunou and Okou (2019) decompose both the actual realized volatility and the option implied volatility into good and bad volatility components and study risk premium associated with good and bad volatility separately. Besides measuring good and bad volatility by summing over higher frequency squared returns directly (Barndorff-Nielsen and Shephard, 2006), Bekaert et al. (2015) develops a parametric model for good and bad “environment” under gamma distributed shocks with time-varying shape parameters. A number of studies also decompose the variance of macroeconomic variables into good and bad components (e.g., Bekaert and Engstrom, 2017; Segal et al., 2015) and find that “good” and “bad” macroeconomic uncertainty have important asset pricing implications.

In a related study, Bollerslev et al. (2020) studies the pricing of a short-lived weekly ex-post realized good minus bad volatility using intraday returns, and hypothesize that either investors rationally price bad volatility more than upside volatility or investors overreact to extreme negative price movements. Differently, we construct a more persistent ex-ante measure of good minus bad volatility and conduct empirical tests to examine relevant economic mechanisms. Our finding suggests that the upside potential is more important and is related to growth options.

The remainder of this paper is organized as follows. Section 2 formally defines good and bad volatility and presents the cross-sectional model used to estimate expected good minus bad volatility. Section 3 shows the main empirical results. Section 4 presents analysis on mechanisms that drive the return predictability of expected good minus bad volatility. Section 5 concludes.

## 2 Measurement of Variables

### 2.1 Realized Good and Bad Idiosyncratic Volatility

Our dataset includes monthly and daily returns data on stocks traded in the NYSE, AMEX, and NASDAQ from the Center for Research in Security Prices (CRSP) merged with the accounting variables from COMPUSTAT's industrial files of income statements and balance sheets. We exclude financial and utility firms with 4-digit Standard Industrial Classification (SIC) codes between 6000 and 6999 and between 4900 and 4999. Following Boyer et al. (2010), we first calculate the residuals of the following time-series regressions using daily observations from month  $t$  to  $t + T - 1$ :<sup>3</sup>

$$r_{i;t;d} = \alpha_{i;t} + \beta_{i;t;MKT} MKT_{t,d} + \beta_{i;t;SMB} SMB_{t,d} + \beta_{i;t;HML} HML_{t,d} + \epsilon_{i;t;d} \quad (1)$$

where for day  $d$  in month  $t$ ,  $r_{i;t;d}$  is stock  $i$ 's excess return, the  $MKT_{t,d}$  is the market excess returns,  $SMB_{t,d}$  and  $HML_{t,d}$  are factors that capture size and book-to-market effects, respectively. And  $\alpha_{i;t}$ ,  $\beta_{i;t;MKT}$ ,  $\beta_{i;t;SMB}$ ,  $\beta_{i;t;HML}$  are regression coefficients. For each stock, we calculate the idiosyncratic realized variance ( $IV$ ), idiosyncratic volatility ( $IVOL$ ), and idiosyncratic skewness ( $ISKEW$ ) using daily residuals  $\epsilon_{i;t;d}$  from the first day of month  $t$  through the end of month  $t + T - 1$ :

$$IV_{i;t} \equiv \sum_{d=1}^{N_m} (\epsilon_{i;t;d})^2; \quad (2)$$

$$IVOL_{i;t} \equiv \sqrt{IV_{i;t} / N_m}; \quad (3)$$

$$ISKEW_{i;t} \equiv \frac{\sqrt{N_m} \sum_{d=1}^{N_m} \epsilon_{i;t;d}^3}{IV_{i;t}^{3/2}}; \quad (4)$$

where  $N_m$  is the total number of trading days between month  $t$  and  $t + T - 1$ . The realized variance measure in equation (2) does not differentiate between volatility that is associated with positive or negative price changes. In this paper, we further decompose the realized variance into “good” and

<sup>3</sup>Results are reported for  $T = 60$  months. We also run analysis for  $T$  ranging from 12 months to 60 months or with more factors. Results are similar and available from the authors.

“bad” components following Barndorff-Nielsen and Shephard (2002):

$$IV_{i;t}^+ = \sum_{d=1}^{N_m} (i;t;d)^2 \mathbf{1}_{\{i,t,d>0\}} \quad (5)$$

$$IV_{i;t}^- = \sum_{d=1}^{N_m} (i;t;d)^2 \mathbf{1}_{\{i,t,d>0\}} \quad (6)$$

where  $IV_{i;t}^+$  and  $IV_{i;t}^-$  are upside and downside realized variance respectively. The upside and downside realized variance measure certainly adds up to the total realized variation,  $IV_{i;t} = IV_{i;t}^+ + IV_{i;t}^-$ . Moreover, following Bollerslev et al. (2020), we define the difference between good and bad idiosyncratic volatility for firm  $i$  at month  $t$  as

$$GMB_{i;t} \equiv \frac{(IV_{i;t}^+ - IV_{i;t}^-)}{IV_{i;t}} \quad (7)$$

The difference between the upside and downside variance is normalized by the realized variance, which naturally removes the overall volatility level from the  $GMB_t$  measure, rendering  $GMB_{i;t}$  to lie between  $-1$  and  $1$ .

The difference between realized upside and downside variance is also known as signed jump variations (Patton and Sheppard, 2015; Bollerslev et al., 2020). Feunou et al. (2013) and Feunou et al. (2016) argue that the difference between realized upside and downside variance are valid estimates for asymmetry in stock return<sup>4</sup>. We complement these studies in examining the relevance of good minus volatility measured over longer horizons for predicting conditional idiosyncratic skewness and stock returns.

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<sup>4</sup>In Appendix, we briefly outline theoretical results that support good minus bad volatility as a measure of the asymmetry of stock returns under mild conditions.



## 2.2 Measuring Expected Good Minus Bad Volatility and Expected Idiosyncratic Skewness

For further asset pricing analysis, we construct measures of expected good minus bad volatility and expected idiosyncratic skewness<sup>5</sup>. Following Boyer et al. (2010) and Bali et al. (2020), we run the following cross-sectional regressions with lagged regressors to obtain regression coefficients<sup>6</sup> and then compute estimates of expected good minus bad volatility (EGMB) and expected idiosyncratic skewness (EISKEW) of month  $t$  using information available to investors at the end of month  $t-1$ :

$$\begin{aligned}
 Z_t = & \alpha + \beta Z_{t-T} + \text{ivol}IVOL_{t-T} + \text{go}GO_{t-1} + \text{ag}AG_{t-1} \\
 & + \text{max}MAX_{t-1} + \text{mom}MOM_{t-1} + \text{turn}TURN_{t-1} + \text{roe}ROE_{t-1} \\
 & + \text{small}SMALL + \text{big}BIG + EXCH + INDU + \epsilon_t
 \end{aligned} \tag{8}$$

In equation (8), the variable  $Z_t$  is a  $M$  by 1 vector of either good-minus-bad volatility  $GMB_t$  or idiosyncratic skewness  $ISKEW_t$  calculated from  $M$  firms' daily residuals from month  $t$  to  $t+T-1$ . The variable  $IVOL_{t-T}$  is realized idiosyncratic volatility estimated using daily residuals from month  $t-T$  and  $t$

a no-further-growth policy from the current market value of the firm  $V$ :

$$GO_{i;t} \equiv \frac{PVGO_{i;t}}{V_{i;t}} = 1 - \frac{CF_{i;t}=V_{i;t}}{k_i}. \quad (9)$$

In equation (9),  $V_{i;t}$  is the market value of firm  $i$  at time  $t$ ,  $CF_{i;t}$  is the operating cash flow of firm  $i$  at time  $t$ , and  $k_i$  is firm  $i$ 's weighted-average cost of capital (WACC). Cash flow is measured as the free operating cash flow under a no-further-growth policy where capital expenditures equal depreciation.<sup>7</sup> To estimate the cost of equity in WACC, we use the market model with beta equal to 1, and we add a 6% market risk premium to the risk-free return for all firms. This simple setup avoids the reliance of our results on the empirical validity of the CAPM. We estimate a firm's cost of debt to be 4% less than its cost of equity. Effective tax rates are tax current (TXC) divided by pretax income (PI). The weighted average cost of capital  $WACC_t$  is then estimated as  $COST\_EQUITY \times (1 - LEV_t) + COST\_DEBT \times LEV_t(1 - )$ , where the leverage  $LEV_t$  is calculated as the ratio of total liabilities to the market value of the firm. The market value of the firm is the sum of market value of equity plus the value of debt approximated by total liabilities ( $LT$ ). Asset growth ( $AG$ ) is calculated as the percent change in firm total assets over the previous year.

We also include several variables used in the literature to predict idiosyncratic skewness. The skewness of stock returns, which may in extreme form manifested as lotteryiness proxied by maximum daily return ( $MAX$ ) during the previous month.  $MAX_{t-1}$  is the maximum daily return observed in month  $t-1$ . Motivated by Hong and Stein (2003) and Chen et al. (2001), turnover and momentum are included in forecasting regressions. Turnover ( $TURN_{t-1}$ ) is calculated as the ratio of trading volume to shares outstanding in month  $t-1$ . Momentum  $MOM_{t-1}$  is the cumulative return over months  $t-12$  through  $t-2$ . The return on equity  $ROE$  is calculated as the ratio of operating cash flow to market value of equity. Variables  $SMALL$  and  $BIG$  are 2 binary dummies for  $SMALL$  (bottom 30%) and  $BIG$  (top 30%) firms built on market capitalization observed in previous month allowing for a non-linear size-skewness relationship as in Boyer et al. (2010). Dummy variables  $INDU$  and  $EXCH$  control for the Fama and French 30 industries and the NASDAQ exchange,

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<sup>7</sup>Under a no-growth policy, capital expenditures roughly equal depreciation. This leads to estimating FCF as

respectively.

### 2.3 Summary Statistics and Determinants of Expected Good Minus Bad Volatility

To corroborate the impact of our main variables on determining expected good minus bad volatility and expected idiosyncratic skewness, we run a series of firm-level cross-sectional regressions based on equation (8)<sup>8</sup>. Panel A of Table 1 reports summary statistics for variables used in our empirical analysis<sup>9</sup>. The market beta (BETA) is close to 1 (Median = 1.05). The mean book-to-market ratio (BM) is 0.78, within the range found in earlier studies (e.g., Anderson and Garcia-Feijoo (2006)). The mean monthly excess return (RET) is 0.98%. The average expected good minus bad volatility (EGMB) across firms has a mean of 0.12 and a standard deviation of 0.07. The average expected idiosyncratic skewness is more volatile. The mean is 0.85 and the standard deviation is 0.70. Panel B of Table 1 reports average Pearson correlation coefficients among the key variables. Growth options (GO), maximum daily returns (MAX), and idiosyncratic volatility (IVOL) are positively associated with expected good minus bad volatility and expected idiosyncratic skewness. Return on equity (ROE), Size (SIZE), and momentum (MOM) are negatively correlated with expected good minus bad volatility and expected idiosyncratic skewness. Expected good minus bad volatility and expected idiosyncratic skewness are strongly correlated, with an average correlation of 0.97.

Panel A of Table 2 shows the time-series averages of cross-sectional slopes using the previously described determinants to predict good minus bad volatility. Growth options (GO), lotteryiness (MAX), and idiosyncratic volatility (IVOL) are statistically significant positive drivers of good minus bad volatility. This result lends support to the idiosyncratic-skewness-enhancing impact of growth options (Bali et al., 2020). Higher values for momentum (MOM), profitability (ROE), investment (AG), and turnover (TURN) are all associated with lower values of GMB. On average, the cross-sectional adjusted- $R^2$  is 0.28. Panel B of Table 2 displays analogous results for idiosyncratic

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<sup>8</sup>To limit the influence of outliers, we winsorize the top and bottom 1% observations for each variable except size.

<sup>9</sup>Because estimating the cross-sectional regressions and obtaining conditional skewness measures (EGMB and EISKEW) requires 10 years of prior returns and relevant accounting data, our subsequent analysis starts from Jan 1978 through December 2020.

skewness. All the determinants show same predicting signs as Panel A, but with a lower average adjusted- $R^2$  of 0.16. These adjusted- $R^2$  numbers are consistent with previous findings (e.g., Chen et al., 2001; Boyer et al., 2010; and Bali et al., 2020).

To illuminate the temporal variation in each of the skewness measures, Figure 1 presents the 10th, 50th, and 90th percentiles of the monthly cross-sectional distributions of realized idiosyncratic skewness (ISKEW), realized good minus bad volatility (GMB), expected idiosyncratic skewness (EISKEW), and expected good minus bad volatility (EGMB), respectively. All of the reported percentiles show substantial time variations, with particularly large movements during the mid-1980s, the early 1990s, the late-1990s, and the early-2010s. Compared to the realized idiosyncratic skewness (Graph A), the realized good minus bad volatility (Graph B) tends to be more stable, with less variation in extreme percentiles, e.g., in the late-1990s. The conditional measures (EISKEW and EGMB) capture similar temporal variation as the realized measures (ISKEW and GMB), but have smaller dispersion between percentiles.

## 2.4 Evaluating the Accuracy of Conditional Idiosyncratic Skewness Measures

We then evaluate the accuracy of expected good minus bad volatility in measuring conditional idiosyncratic skewness. Based on theoretical results from Feunou et al. (2016), we use both realized good minus bad volatility ( $GMB_t$ ) and realized idiosyncratic skewness ( $ISKEW_t$ ) calculated using daily returns from  $t$  to  $t + T - 1$  as proxies for ex post skewness. To be specific, in each month  $t$ , we evaluate conditional skewness measures  $\widehat{Z}_t$ , which are computed from (8) or simply equal to lagged skewness  $Z_{t-T}$ , in forecasting realized idiosyncratic skewness  $Z_t$ .

Panel A of Table 3 reports the average cross-sectional Pearson correlations and Spearman rank correlations between realized skewness measures and conditional forecasts. Lagged skewness values ( $ISKEW_{t-T}$  and  $GMB_{t-T}$ ) show low correlations with ex post realized values, ranging from 0.2 to 0.3. The expected measures (EISKEW and EGMB) outperform historical values in forecasting realized skewness by quite a wide margin, with average correlations about 0.45. This result highlights the usefulness of the cross-sectional regression approach to estimate conditional skewness. (e.g., Boyer et al., 2010 and Bali et al., 2020). Furthermore, EGMB shows a higher correlation with

future realized values than *EISKEW*. For example, the correlation between *EGMB<sub>t</sub>* and *GMB<sub>t</sub>* is 0.46, while the correlation between *EISKEW<sub>t</sub>* and *GMB<sub>t</sub>* is 0.43. For realized idiosyncratic skewness measure *ISKEW<sub>t</sub>*, *EGMB<sub>t</sub>* also yields the highest correlation.

To further examine forecasting accuracy of different conditional idiosyncratic skewness measures, we conduct analysis using root mean squared error (RMSE), a loss function commonly used in the literature <sup>10</sup>. Since these estimators lie in different ranges, we transform the realized values and predicted values into their cross-sectional ranks to facilitate comparisons. The RMSE at month *t* is defined as:

$$RMSE_t = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (Rank(Z_{i;t}) - Rank(\hat{Z}_{i;t}))^2}; \quad (10)$$

where  $Z_{i;t}$  is realized good minus bad volatility  $GMB_{i;t}$  or realized idiosyncratic skewness  $ISKEW_{i;t}$  for stock  $i$  and  $\hat{Z}_{i;t}$  denotes conditional forecasts (e.g.  $EGMB_t$  or  $GMB_{t-\tau}$ ). The  $Rank(Z_{i;t})$  function gives the order  $(1, 2, \dots, N_t)$  of variable  $Z_{i;t}$  in all values  $Z_t$  sorted in ascending orders and  $N_t$  is the number of stocks available at time  $t$ . In each month  $t$ , we test for significance in RMSE differences using the modified Diebold-Mariano test proposed by Harvey et al. (1997).

The first row in Panel B of Table 3 presents average out-of-sample RMSEs of different estimators for predicting  $GMB_t$ . Among all conditional measures, expected good minus bad volatility (EGMB) achieves the lowest RMSE, while lagged skewness is the least accurate predictor. The RMSE values are 585.46 and 680.90, respectively. For pairwise comparisons between different measures, the average RMSE of EGMB, for example, is lower than EISKEW by 14.61 and the difference between these two methods is significant at 5% level 80% of the time. The average RMSE of EGMB is significantly lower than that of  $GMB_{t-\tau}$  and  $ISKEW_{t-\tau}$  almost all the time (98%).

More importantly, it should be noted from Panel C of Table 3 that the expected good minus bad volatility  $EGMB$  is also the most accurate predictor for realized idiosyncratic skewness ( $ISKEW_t$ ). Panel C of Table 3 conducts similar analysis for predicting  $ISKEW_t$ . Among all considered conditional skewness measures, EGMB still yields the lowest prediction errors with an

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<sup>10</sup>The results are also robust to using mean absolute error (MAE) loss function.

average RMSE of 611.51, while lagged skewness has the worst performance with an RMSE of 693.96. As for the difference between these methods, although the leading edge of EGMB over EISKEW slightly narrows, the difference is still statistically significant 56% of the time.

To sum up, both the correlation and prediction errors analysis provide evidence that expected good minus bad volatility is the most precise predictor of conditional idiosyncratic skewness. The finding that this good-minus-bad-volatility-based measure provides superior conditional idiosyncratic skewness forecast is novel in the literature. While Feunou et al. (2016) and Bollerslev et al. (2020) have shown the relevance of good minus bad volatility to realized skewness using high-frequency intra-day data, we highlight that the importance of good minus bad volatility for measuring conditional idiosyncratic skewness over longer horizons for a broader cross-section of firms.

### **3 Expected Good Minus Bad Volatility and Future Stock Returns**

In this section, we first analyze the relationship between expected good minus bad volatility (EGMB) or expected idiosyncratic skewness (EISKEW) and future stock returns using portfolio analysis. Subsequently, we investigate the cross-sectional relationship between expected good minus bad volatility and stock returns using firm-level Fama and MacBeth (1973) regressions that simultaneously control for expected idiosyncratic skewness (EISKEW) and other firm characteristics.

#### **3.1 Univariate Portfolio Analysis**

We now examine the cross-sectional relationship between expected good minus bad volatility (EGMB) or expected idiosyncratic skewness (EISKEW) and stock returns using univariate portfolio analysis. At the end of each month, value-weighted quintile portfolios are formed by sorting stocks based on EGMB or EISKEW, where quintile 1 portfolio contains stocks with the lowest EGMB or EISKEW and quintile 5 portfolio contains stocks with the highest EGMB or EISKEW. We also report the results for a self-financing long–short (5-1) portfolio that buys stocks in the top quintile and sells stocks in the bottom quintile.

Panel A of Table 4 reports, by row, the time-series average of EGMB, the one-month ahead excess returns and respective Newey-West adjusted standard errors, market shares, and the risk-adjusted alphas for constructed portfolios. The average monthly excess return decreases monotonically from 0.81% for quintile 1 (Low) to  $-0.09\%$  for quintile 5 (High). The difference in monthly returns between the highest and lowest EGMB portfolio is  $-0.91\%$ , with a  $t$ -statistic of  $-2.82$ . We also control for exposure to systematic risk factors. The FF3 alpha is returns in excess of the market (MKT), size (SMB), and book-to-market (HML) factors of Fama and French (1993) (FF3).  $q$ -factor alpha is excess returns to Hou et al. (2015)  $q$ -factor model. FF5 alpha is excess returns relative to the Fama and French (2015) 5-factor (FF5) model, which is FF3 model augmented with the investment factor (CMA) and profitability factor (RMW). The results in panel A show that alpha spreads between quintiles 5 and 1 adjusted by FF3,  $q$ -factor, and FF5 models are all negative and significant:  $-1.32\%$  per month for FF3 model,  $-0.47\%$  for the  $q$ -factor model, and  $-0.72\%$  for the FF5 model with robust  $t$ -statistics of  $-5.01$ ,  $-2.34$ , and  $-3.68$  respectively.

Panel B of Table 4 displays monthly portfolio returns of sorting on EISKEW. The performance of EISKEW sorted portfolios confirms the findings of Boyer et al. (2010) and Bali et al. (2020). In particular, the average monthly return decreases monotonically from 0.81% for quintile 1 (Low) to 0.17% for quintile 5 (High). The long-short EISKEW portfolio generates an average monthly return of  $-0.63\%$ , with a  $t$ -statistic of  $-2.19$ . These numbers are quantitatively very similar to the finding of Boyer et al. (2010) that the expected idiosyncratic skewness sorted long-short (5–1) portfolio earns an average of  $-0.67\%$  per month from December 1987 through November 2005. Panel B of Table 4 also shows that the risk adjusted alphas of EISKEW sorted portfolios are all negative, but smaller than alphas of EGMB sorted portfolios.

While both EGMB and EISKEW sorted portfolios yield statistically significant return spreads, the magnitude of EGMB long-short portfolio ( $-0.91\%$  per month) is larger than EISKEW long-short portfolio ( $-0.63\%$  per month). Risk-adjusted alphas using different factor models also show this pattern. For example, the FF3 adjusted alpha of EGMB sorted portfolio is  $-1.32\%$  per month, while EISKEW sorted portfolios is  $-0.99\%$ . To further illuminate how important these differences are, Figure 2 depicts the cumulative profits for the strategies based on long-short portfolios of

buying stocks with low conditional skewness and shorting stocks with high conditional skewness.

Start with an initial investment of  $W_0 = \$1$ , the cumulative profits in month  $t$  are calculated as:

$W_t = W_{t-1} \times (1 + r_{long-short;t} + r_{f;t})$ , where  $r_{long-short;t}$  denotes the monthly return of the long low

EGMB/EISKEW and short high EGMB/EISKEW portfolio and  $W$



and profitability (Langlois, 2020).

The last two rows in Panel A and B in Table 4 present portfolio sort results after the inclusion of FEISKEW and FEGMB. When we augment FF5 model with the FEISKEW factor, the long-short portfolio sorted by EGMB still shows statistically significant abnormal return, i.e.  $-0.47\%$  per month ( $t$ -statistics =  $-3.21$ ). On the contrary, after controlling for the expected good minus bad volatility factor (FEGMB), the risk-adjusted return spreads are economically and statistically insignificant for EISKEW sorted portfolio:  $-0.27\%$  per month ( $t$ -statistics =  $-1.52$ ). These results suggest that the expected good minus bad volatility factor has some power to explain the return spreads between portfolios sorted by expected idiosyncratic skewness. However, the expected idiosyncratic skewness factor is not sufficient to explain the negative relationship between expected good minus bad volatility and future stock returns <sup>11</sup>.

### 3.3 Double-Sorted Portfolios

The results from univariate portfolio sorts in Tables 4 reveal a strong negative relation between expected good minus bad volatility and future stock returns. And such relation cannot be explained by exposure to previously documented pricing factors and the expected idiosyncratic skewness effect. By contrast, the negative relation between expected idiosyncratic skewness and future stocks becomes insignificant after controlling for expected good minus bad volatility factor (FEGMB). In this section, we continue our examination of the superior performance of expected good minus bad volatility using bivariate-sort portfolio analysis. Specifically, we focus on whether the negative relation between expected good minus bad volatility and future stock returns persists in bivariate portfolio analysis controlling for expected idiosyncratic skewness, and vice versa.

Panel A of Table 6 shows the results of sorting on EGMB after first sorting on EISKEW. For each month, all stocks in the sample are sorted into five quintiles based on ascending order of EISKEW. Within each EISKEW quintile, we then sort stocks into quintiles based on EGMB and compute the value-weighted one-month-ahead return for the resulting 25 ( $5 \times 5$ ) portfolios. To focus

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<sup>11</sup>In Appendix B.3, we provide analysis for the reasoning that expected good minus bad volatility has more explanatory power than expected idiosyncratic skewness because it measures conditional idiosyncratic skewness more accurately.

more directly on the effect of EGMB, we also compute returns averaged across EISKEW quintiles as a way to produce quintile portfolios with large variations in EGMB, but small variations in EISKEW. These returns are reported in the last row labeled “Avg”. The column labeled “5–1” reports the difference in returns between high and low EGMB portfolios within each EISKEW quintile. The column labeled “FF3 alpha” reports the average Fama–French 3-factor alphas.

The negative relationship between expected good minus bad volatility and future monthly returns can be observed in all five EISKEW sorted quintiles, especially in the high EISKEW groups (e.g. the third to fifth EISKEW quintiles). On average, the resulting FF3 alpha for the difference in returns between low and high EGMB quintile portfolios obtained by first sorting on EISKEW equals  $-0.68\%$ . This economically large spread also has a highly significant  $t$ -statistic of  $-4.35$ , again underscoring that EISKEW cannot explain the return predictability of EGMB.

Panel B of Table 6 presents the results from sequential double sorts, in which we first sort on EGMB and then on EISKEW. After controlling for expected good minus bad volatility, the negative relationship between EISKEW and one-month-ahead returns shown in single portfolio sorts (Table 4) diminished and even reversed. Except for the first quintile of EGMB, all the return spreads between the lowest and highest quintile portfolios sorted by EISKEW are insignificant. On average, the FF3 alpha for the return spread between value-weighted portfolios sorted by EISKEW equals  $-0.09\%$ , with a statistically insignificant  $t$ -statistic of  $-0.73$ .

In sum, the negative relation between expected good minus bad volatility and future returns remains significant after controlling for expected idiosyncratic skewness. On the contrary, the return predictability of expected idiosyncratic skewness completely disappears, and even reverses, when controlling for the expected good minus bad volatility. Our results highlight that expected good minus bad volatility may capture more asset pricing relevant information than expected idiosyncratic skewness.

### **3.4 Firm-Level Fama-MacBeth Cross-Sectional Regressions**

Previous sections provide evidence of a strong negative relation between expected good minus bad volatility and cross-sectional stock returns using portfolio analysis. And the relation cannot

be explained by exposure to systematic risk factors and expected idiosyncratic skewness. In this section, we examine that relation using Fama and MacBeth (1973) regressions. In each month, we run cross-sectional regressions of one-month-ahead excess stock returns on expected good minus bad volatility and various control variables:

$$r_{i;t+1} = \alpha_{0;t} + \beta_{1;t}EGMB_{i;t+1} + \gamma_{i;t+1}'X_{i;t} \quad (11)$$

where  $r_{i;t+1}$  is the monthly excess return for stock  $i$  observed at the end of month  $t+1$ ,  $EGMB_{i;t+1}$  is the expected good minus bad volatility for stock  $i$  of month  $t+1$  based on information available at month  $t$ , and  $X_{i;t}$  is a set of firm-specific control variables observed at time  $t$  for stock  $i$ .

Table 7 reports the time-series averages of the  $\alpha$  and  $\beta$  coefficients along with Newey and West (1987) standard error adjusted  $t$ -statistics. Row 1 reports the cross-sectional pricing of expected good minus bad volatility after controlling for the market BETA, market capitalization (SIZE), and book-to-market (BM). The coefficient on expected good minus bad volatility is negative and significant at the 1% level with a robust  $t$ -statistic of  $-4.47$ . Row 2 reports similar results for expected idiosyncratic skewness, in which the coefficient is significant with  $t$ -statistic  $-4.72$ . Row 4 reports the result when both EGMB and EISKEW are simultaneously included in the regression. After controlling for the effect of EISKEW, the negative cross-sectional relation between EGMB and future stock returns remains significant with a  $t$ -statistic of  $-2.04$ . On the other hand, the EISKEW effect is completely reversed with positive coefficients. This finding mirrors the results based on the double sorts in Section 3.3 <sup>12</sup>.

Rows 5 to 10 further control for other characteristics including growth options (GO), momentum, profitability (ROE), investment (AG), idiosyncratic volatility (Ang et al., 2006, IVOL), co-skewness (Harvey and Siddique, 2000, COSKEW), and lottery preference (Bali et al., 2011, MAX). All regression results show that coefficient on expected good minus bad volatility remains significant, which confirms that the return predictability of expected good minus bad volatility (EGMB) can not be subsumed by common return predictors.

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<sup>12</sup>In Appendix B.3, we provide evidence that the change of sign is consistent with the reasoning that EGMB provides a more accurate measure of conditional idiosyncratic skewness.

## 4 Explaining the Relationship between Good Minus Bad Volatility and Stock Returns

To gain a better understanding of the findings of this paper, we investigate economic mechanisms that drive the cross-sectional relationship between expected good minus bad volatility and future stock returns. First, we investigate whether the stronger return predictability of expected good minus bad volatility (EGMB) comes from measuring conditional idiosyncratic skewness more accurately. Second, we examine whether recently proposed skewness-related factors could explain the return predictability of expected good minus bad volatility. Third, we relate our findings to growth option theory. Finally, we investigate whether aversion to downside volatility plays an important role in the return predictability of expected good minus bad volatility.

### 4.1 The Role of Forecast Accuracy

In previous sections, we find that expected good minus bad volatility (EGMB) is a more accurate measure of conditional skewness, and yields stronger return predictability than expected idiosyncratic skewness. If a negative relationship between unobserved conditional idiosyncratic skewness and stock returns exist, expected good minus bad volatility, which has less measurement errors in measuring conditional idiosyncratic skewness, could have stronger return predictability than expected idiosyncratic skewness.

To validate this argument, we divide the full sample into subsamples based on the Diebold-Mariano statistics (DM hereafter) calculated in Section 2.4. For each month  $t$ , this DM statistic evaluates the significance of differences between EGMB and EISKEW in forecasting ex-post idiosyncratic skewness. A higher DM statistic indicates that EGMB is relatively more accurate than EISKEW in forecasting conditional idiosyncratic skewness.

To test for the effects of forecast accuracy on return predictability, we characterize months into periods of high (low) relative forecast accuracy if the DM statistic is above (below) the median. In Table 8, we summarize value-weighted returns and factor-model adjusted alphas for long-short portfolios that buy stocks in the lowest quintile of EGMB (or EISKEW) and sell those in the highest

quintile in subsamples. The “Diff” column reports the return differences between the EGMB and EISKEW long-short portfolios.

The results in Table 8 indicate that the EGMB strategy outperforms the EISKEW strategy, especially during periods when EGMB accurately predicts future skewness. In the “High” relative forecast accuracy periods, the differences between the alphas on the EGMB and EISKEW long-short portfolios are large, ranging from 0.31 to 0.34 percent, and are highly significant. In contrast, when the forecast accuracy of conditional idiosyncratic skewness is similar, differences between these two strategies are small and statistically insignificant. These results confirm our hypothesis that the superior return predictability of the EGMB strategy arises from its better forecasting ability for conditional idiosyncratic skewness.

## 4.2 Controlling for Skewness-Related Factors

Recent studies (Barinov, 2018; Langlois, 2020) suggest that firms with high idiosyncratic skewness may offer hedges to skewness-related factors, and hence earn low returns. To be specific, Barinov (2018) shows that lottery-like stocks are a hedge against increases in aggregate volatility risk thus earning low expected returns. He constructs an aggregate volatility risk factor (FVIX) and shows that the maximum effect of Bali et al. (2011) and the expected skewness effect of Boyer et al. (2010) could be reduced after incorporating the FVIX factor. Langlois (2020) argues that the cross-sectional ranks of systematic and idiosyncratic skewness other than their actual values are easier to predict and proposes the predicted systematic skewness (PSS) factor. He shows that the abnormal returns of idiosyncratic skewness sorted portfolio diminished to insignificant levels after the PSS factor is included.

We investigate whether the return spread between portfolios sorted by EGMB or EISKEW can be explained by exposure to FVIX and PSS factors. Table 9 presents the risk-adjusted returns (alphas) controlling for FVIX or PSS factors<sup>13</sup>. Row “FF5+FVIX” in Panel A and B of Table 9 shows that the inclusion of FVIX factor besides FF5 factors has little impact on the return spreads

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<sup>13</sup>The PSS factor data are collected from Langlois' website (<https://hugueslanglois.com/>) and we construct the FVIX factor following Barinov (2018). The FVIX factor is only available after 1986 when the VXO index was introduced. As a test of validity, the correlation between our FVIX factor and  $VXO$  is 0.719, which is almost equal to the correlation of 0.715 reported in Table 1 of Barinov (2018)

between lowest and highest quintile portfolios sorted by EGMB or EISKEW. The spread is still economically and statistically significant: from  $-0.61\%$  for FF5 model to  $-0.59\%$  per month for EGMB and  $-0.55\%$  for FF5 model to  $-0.57\%$  for EISKEW, respectively. Thus the FVIX factor cannot explain the return predictability of expected good minus bad volatility. The last row in Panel A and B of Table 9 presents the results of augmenting the FF5 factor model with PSS factor. The risk-adjusted return spread of EGMB sorted portfolios slightly is reduced to  $-0.51\%$  and is still significant with  $t$ -statistic  $-2.38$ . In contrast, the alpha of EISKEW sorted portfolio is marginal significant with  $t$ -statistics  $-1.77$ . Thus, the return predictability of idiosyncratic skewness may be to some extent explained by the predicted systematic skewness factor, which is in line with the findings of Langlois (2020).

To sum up, recent studies (Barinov, 2018 and Langlois, 2020) suggest that idiosyncratic skewness *per se* are less relevant for future returns, because stocks with high idiosyncratic skewness offer hedges to some risk factors and hence earn low returns. Our results demonstrate that the strong return predictability of expected good minus volatility remains after controlling for these skewness-related factors. Thus, our results emphasize the importance of conditional idiosyncratic skewness in portfolio construction and return diversification.

### 4.3 The Role of Growth Options

Various studies have investigated the importance of growth options (e.g., Berk et al., 1999, Cao et al., 2008; Grullon et al., 2012 and Trigeorgis and Lambertides, 2014) in affecting expected returns and risk characteristics. A fundamental property of growth options and real options in general (besides becoming more valuable in volatile environment) is their discretionary asymmetric nature: they are rights but not obligations whose exercise provides valuable firm flexibility. Del Viva et al. (2017) posit that actively managed firms have real options leading to convex payoffs since protective contraction put options reduce downside risk, whereas growth options preserve and enhance upside potential. As a result, this dynamic asset adaptation process creates a convex value payoff that enhances idiosyncratic skewness. Recent theories also concur (e.g, Brunnermeier and Parker, 2005; Mitton and Vorkink, 2007 and Barberis and Huang, 2008) that investors may have preferences for

idiosyncratic skewness. Such investor preferences predict that high idiosyncratic skewness stocks have lower expected returns in equilibrium.

Row 3 of Table 7 confirms that growth option intensity (GO) exhibits a significant negative relation with subsequent stock returns, which is consistent with evidence from existing studies (e.g., Anderson and Garcia-Feijoo, 2006 and Trigeorgis and Lambertides, 2014). When expected good minus bad volatility (EGMB) and growth options (GO) are included in the regression simultaneously, it is important to note from row 5 of Table 7 that the coefficient on growth option (GO) becomes insignificant. This finding complements the empirical evidence of Bali et al. (2020). While Bali et al. (2020) emphasize the part of expected idiosyncratic skewness that is only related to growth options (GO) in explaining several return anomalies, we show that the predictability

where  $BV_{i;t}$  is the realized bipower variation (see Barndorff-Nielsen and Shephard (2006) or appendix.) We thus define the “good jump” component and “bad jump” component as<sup>14</sup>:

$$GJ_{i;t} \equiv \frac{(IV_{i;t}^+ - BV_{i;t=2})}{IV_{i;t}} \quad (12)$$

$$BJ_{i;t} \equiv \frac{(BV_{i;t=2} - IV_{i;t}^-)}{IV_{i;t}}. \quad (13)$$

The GJ component mainly captures the return asymmetry caused by positively skewed return realizations, while the BJ component is associated with negative skewed return realization. After extracting these two components, we follow the same methods in Section 2 to compute expected good jump (EGJ) and expected bad jump (EBJ) using Equation (8).

Figure 3 presents the 10th, 50th, and 90th percentiles of monthly cross-sectional distributions for four measures: realized good jump (GJ), realized bad jump (BJ), expected good jump (EGJ), and expected bad jump (EBJ). Over the sample period, the median of GJ ranges from 0.1 to 0.2, while the median of bad jump is close to zero. This suggests that, on average, individual stock returns are positively skewed, which aligns with previous findings (e.g. see Jondeau et al., 2019). Moreover, both realized and expected measures of good jumps exhibit higher cross-sectional variations than those of bad jumps. Numerically, the time-series averages of cross-sectional standard deviation of EGJ and EBJ are 0.053 and 0.031, respectively, indicating that good jumps play more roles in capturing the cross-sectional variation of good minus bad volatility.

To test for return predictability of expected good and bad jumps, we conduct a series of Fama and MacBeth (1973) regressions like Section 3.4 and Table 10 presents the results. Rows 1 to 3 report the regression coefficients of expected good and bad jumps after controlling for the standard Fama-French 3 factors. The results show that higher EGJ or EBJ significantly predict lower future returns, which means both good jumps and bad jumps are important for the return predictability of EGMB. Rows 4 to 7 analyze the relationship between growth options and stock returns, after controlling for expected good (bad) jumps. When expected good jump is included in the regression, the coefficient of GO is diminished from -0.095 to -0.061 and becomes insignificant. However, when

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<sup>14</sup>In Appendix, we briefly outline theoretical results that support the decomposition of expected minus bad volatility.



we only control for expected bad jump, the pricing ability of GO is reduced to  $-0.078$  but still with a significant  $t$ -statistic of  $-1.98$ . Therefore, these empirical results suggest that growth options play more roles in enhancing a firm’s upside return potentials than reducing downside losses, and this skewness channel is important to explain why growth options earn lower returns. Rows 8 to 12 further control for other characteristics, and the coefficients of expected good jump and expected bad jump remain negative and significant. Thus the predictability of good and bad jumps can not be explained by previously documented factors.

While Bollerslev et al. (2020) hypothesize that the return predictability of a weekly realized good minus bad volatility may arise as investors overreact to extreme negative returns, this line of reasoning doesn’t gain much empirical support in our study. Even though the regression coefficient is slightly higher for expected bad jump component, the overall impact of EBJ on returns is smaller, given the relatively small magnitude of cross-sectional variation of EBJ. As a comparison, a one-standard-deviation increase in EGJ and EBJ predicts a decrease of  $(0.053 \times 6.041 \approx) 0.32\%$  and  $(0.031 \times 9.099 \approx) 0.28$  in monthly returns, respectively, indicating that EGJ could have more power in explaining cross-sectional stock returns. Furthermore, in Appendix B.6, we conduct portfolio sorts to investigate the return predictability of each component. We find that the predictive power of either component alone is significantly weaker compared to EGMB. Hence, our results imply that both good and bad jump components are important for the return predictability of EGMB, which is not solely driven by downside risk (e.g., Gul (1991) and Ang et al. (2006)).

#### 4.5 Macroeconomic Good and Bad Uncertainty

A few papers (e.g. Segal et al., 2015; Bekaert and Engstrom, 2017) investigate the implications of macroeconomic “good” and “bad” uncertainty for asset prices. By decomposing the volatility of macroeconomic series into good and bad components, Segal et al. (2015) find that good uncertainty, which is associated with positive innovations to macroeconomic growth, has a positive effect on economic activity and leads to higher stock prices. In contrast, bad uncertainty, which is associated with negative innovations to macroeconomic growth, has a negative effect on these variables. In this section, we investigate whether macroeconomic good and bad uncertainty are useful to explain

this paper’s empirical finding on growth options.

Following Segal et al. (2015), we construct expected consumption growth ( $CG_t$ ) and ex ante good and bad uncertainty ( $V_g$  and  $V_b$ )<sup>15</sup>. Figure 4 presents the log realized semi-variances and the ex ante macroeconomic uncertainties in our sample period. Similar to Segal et al. (2015), we find that good uncertainty generally goes down in bad times while bad uncertainty increases in recessions.

For cross-sectional tests, we measure the risk exposure of each stock to good and bad uncertainty using the method proposed by Fama and French (1992). Specifically, in each year  $t$ , we estimate pre-ranking  $\beta$ s using data from previous five years:

$$r_{i;t} = \alpha_i + \beta_{i;cg}CG_t + \beta_{i;gv}V_{g;t} + \beta_{i;bv}V_{b;t} + \epsilon_{i;t} \quad (14)$$

where  $r_{i;t}$  is annual excess returns for stock  $i$  in year  $t$ , and the expected consumption growth  $CG_t$  as in Segal et al. (2015) is used as a control. Next, we construct portfolios in a  $5 \times 5 \times 5$  grouping based on the quintiles of market capitalization ( $SIZE_i$ ), good uncertainty beta ( $\beta_{i;gv}$ ), and bad uncertainty beta ( $\beta_{i;bv}$ ). Then, we estimate post-ranking  $\beta$ s using the full sample of each portfolio’s value-weighted returns from equation (14) and assign post-ranking portfolios to stocks in these portfolios.

To investigate whether exposures to macroeconomic uncertainty could explain the return patterns of growth options, we conduct Fama-MacBeth regressions as in Section 3.4 and control for stocks’ risk exposures  $\beta_{gv}$  and  $\beta_{bv}$ . Table 11 presents the time-series averages of estimated coefficients and Newey and West (1987) adjusted  $t$ -values. Consistent with the findings of Segal et al. (2015), we observe a significant negative market price of bad macroeconomic uncertainty, while the slope coefficient of good uncertainty is significantly positive. Nevertheless, controlling for the risk exposure to  $V_g$  and  $V_b$  has little impact on the predictive ability of GO. In the last row of Table 11, we include EGMB in the regression. And in line with our previous findings, the coefficient for GO becomes insignificant. Therefore, these results indicate that exposure to macroeconomic uncertainty doesn’t capture the role of expected good minus bad volatility in explaining the return

<sup>15</sup>Appendix B.7 describes our construction in detail.

patterns of growth options.

## 4.6 Skewness and Market Sentiment

Barberis and Huang (2008) posits that errors in the probability weighting of investors cause them to overvalue stocks with idiosyncratic skewness. Hence, stocks with higher idiosyncratic skewness earn lower future returns. The empirical results of this paper are consistent with their argument. Meanwhile, Stambaugh et al. (2012) suggest that in the presence of short-selling constraints, anomalies associated with overpricing will be much stronger following periods of high market-wide investor sentiment. If the expected idiosyncratic skewness anomaly is driven by the overpricing of high idiosyncratic skewness stocks, we would expect its long-short strategy to be more profitable when market sentiment is high.<sup>16</sup>

Following Stambaugh et al. (2012), we obtain the monthly market-wide investor sentiment index constructed by Baker and Wurgler (2006) and investigate whether sentiment predicts returns of skewness-related anomalies using predictive regressions. Specifically, we estimate the following time-series regression:

$$R_{i,t} = a + b_i * S_{t-1} + u_t; \tag{15}$$

where  $R_{i,t}$  is the excess return of EGMB (or EISKEW) quintile portfolios  $i$  of month  $t$ ,  $S_{t-1}$  is the previous month's level of the investor-sentiment index of Baker and Wurgler (2006),  $u_t$  is the residual term. Consistent with Baker and Wurgler (2006) and Stambaugh et al. (2012), Panel A of Table 12 reports negative coefficients of the sentiment index ( $b$ ) for all quintile portfolios. Moreover, the regression coefficient of long-short portfolio (5–1) is significantly negative, suggesting that the EGMB and EISKEW anomalies are stronger following periods of high market sentiment. For example, the slope of  $-2.33$  on EGMB long-short portfolio implies that a one-standard-deviation

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<sup>16</sup>Recent studies (e.g., Huang et al. (2015); Jiang et al. (2019)) also highlight the importance of sentiment in predicting aggregate or cross-sectional stock returns. In untabulated results, we also estimate sentiment betas and incorporate them into the skewness prediction Equation (8). However, we did not find a significant improvement in predictive power, as indicated by the negligible increase in  $R^2$ . While proxies for individual stock sentiment might potentially enhance skewness prediction, we don't think they are critical to the results of this paper and leave this direction for future research.

increase in sentiment predicts a rise of approximately  $(2.33 \times 0.67 \approx) 1.56\%$  in the long-short portfolios' monthly returns. And the difference in long-short portfolio's return is primarily driven by the lower returns of the short leg (i.e. Highest EGMB or EISKEW portfolio). In Panel B of Table 12, we further control for the Fama and French (2015) 5 factors in the regression. The results remain qualitatively similar.

Overall, our findings suggest that market sentiment reinforces the idiosyncratic skewness anomaly, which is consistent with the overpricing-based explanations of idiosyncratic skewness anomalies.

## 5 Conclusion

We decompose realized idiosyncratic volatility into good and bad volatility and estimate a cross-sectional model for expected good minus bad volatility. The expected good minus bad volatility not only measures conditional idiosyncratic skewness more accurately than expected idiosyncratic skewness and lagged skewness measures, but also yields superior cross-sectional return predictability.

The return predictability of expected good minus bad volatility is also robust to potential risk factors and control variables. Of particular interest, the aggregate volatility risk factor (Barinov, 2018) and predicted skewness factor (Langlois, 2020) cannot explain the return spreads between portfolios sorted by expected good minus bad volatility, while they may reduce unexplained return spreads between portfolios sorted by expected idiosyncratic skewness to insignificant levels. We interpret our results as highlighting the context of a market with underdiversified investors, who have a preference for occasionally large positive gains. In fact, it may be the preference for skewed payoffs that lead to underdiversification in equilibrium (Mitton and Vorkink, 2007).

Further empirical tests suggest that the stronger return predictability of expected good minus bad volatility is consistent with expected idiosyncratic volatility being a more accurate measure of conditional idiosyncratic skewness. We also find that expected good minus bad volatility may play an important role in explaining the negative relationship between growth option and cross-sectional stock returns. When expected good minus bad volatility is included in the regression, growth

option intensity, i.e., the proportion of firm's value that derives from future growth opportunities, becomes insignificant to predicts future stock returns. Our finding lends support to real option theory, which posits that flexibility of growth options may enhance firm's upside value potential and reduces downside risk in bad economic states. Such asymmetric effect of growth options could result in positively skewed returns, which are attractive to investors with a preference for skewness.

The distinction between good minus bad volatility also allows us to separately study the contribution of good volatility and bad volatility in driving the cross-sectional relation between expected good minus bad volatility and future stock returns. Although investors may display strong aversion to extreme negative returns, we don't find strong evidence for such preference, at least for idiosyncratic returns.

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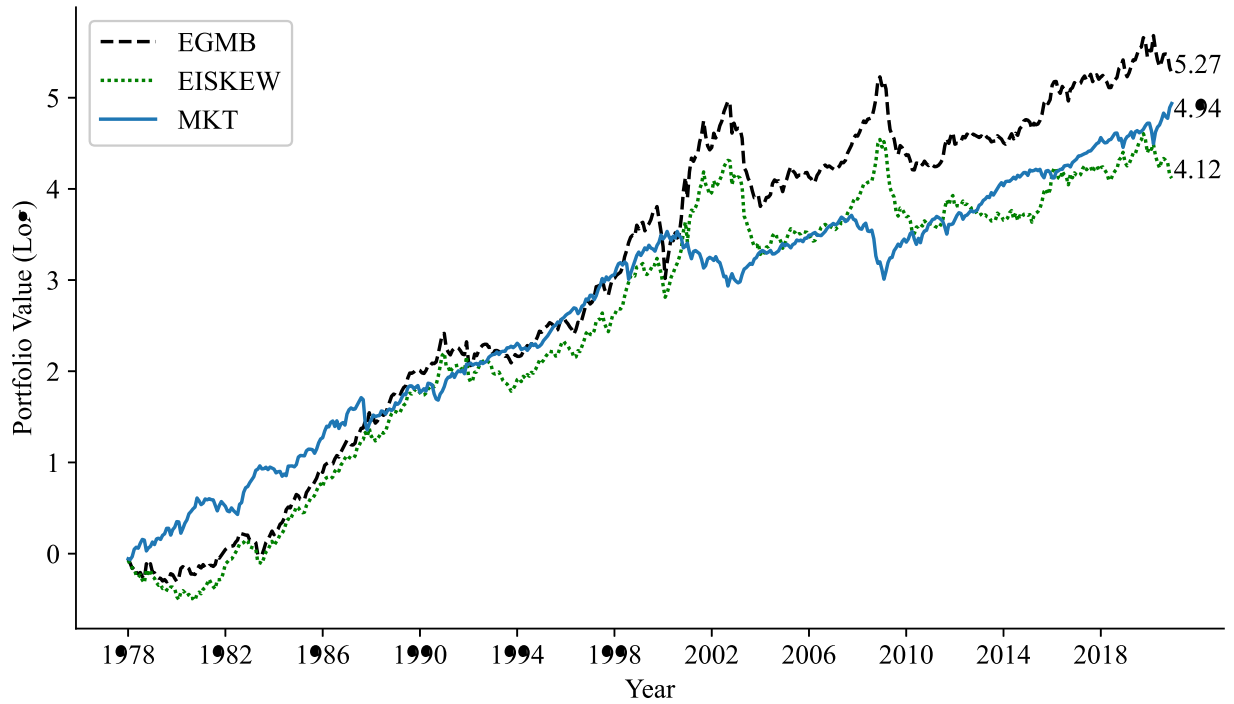


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Figure 1: Cross-sectional Distribution of Firm-level Skewness Measures

Figure 2: Cumulative Portfolio Gains



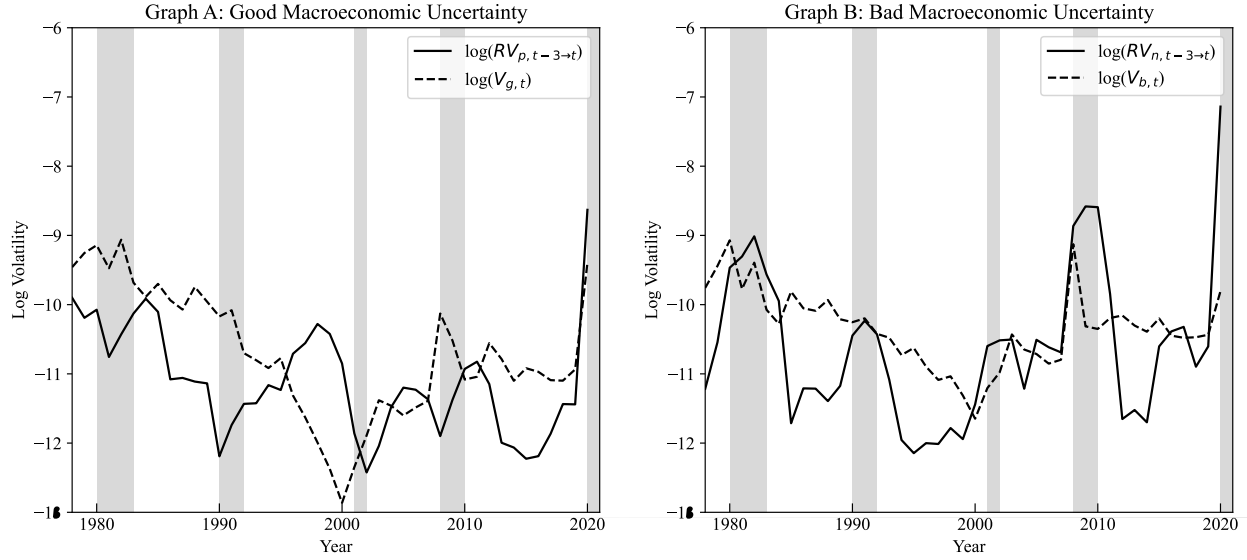
This figure shows the log cumulative portfolio value for the MKT factor and value-weighted long-short portfolio based on the expected good minus bad volatility (EGMB) and expected idiosyncratic skewness (EISKEW) from Jun 1978 to December 2020. The final log portfolio value is 5.27 for EGMB strategy, 4.12 for EISKEW strategy, and 4.94 for MKT factor, respectively. All the portfolios are rebalanced and accumulated on a monthly basis, as described in the main text.

Figure 3: Cross-sectional Distribution of Firm-level Jump Components



Graphs A to D display the 10th, 50th, and 90th percentiles of the monthly cross-sectional distribution of realized good jump (GJ), realized bad jump (BJ), expected good jump (EGJ), and expected bad jump (EBJ), respectively.

Figure 4: Good and Bad Macroeconomic Realized Uncertainty



The figure shows the time series plot of logarithm of good (Graph A) and bad (Graph B) realized semi-variances smoothed over a 3-year window and their ex ante values from the projection. The good and bad realized semi-variances are based on the sum of squared observations of positive and negative movements in industrial production growth over 1-year window, respectively. The ex ante good and bad uncertainties are the projection based on Equation (B.13). The shaded areas represent NBER recessions.

Table 1: Summary Statistics of Main Variables

*Panel A. Summary Statistics*

	RET	BETA	SIZE	BM	MOM	AG	ROE	LEV	GO	MAX	TURN	IVOL	EISKEW	EGMB
Mean	0.98	1.16	12.05	0.78	0.13	0.21	0.05	0.37	0.81	0.08	1.31	3.52	0.85	0.12
Median	-0.20	1.05	11.96	0.57	0.04	0.04	0.11	0.34	0.53	0.06	0.85	3.13	0.70	0.11
Std	17.12	0.68	2.06	0.82	0.54	0.68	0.61	0.23	1.63	0.07	1.55	1.81	0.71	0.07
Min	-72.30	-0.16	7.78	-0.85	-0.76	-0.46	-3.21	0.01	-2.75	0.01	0.04	1.11	-0.78	-0.05
Max	276.88	3.42	17.11	4.73	2.47	5.11	2.81	0.92	8.52	0.43	9.71	10.11	4.00	0.41

*Panel B. Cross Correlations of Main Variables*

	RET	BETA	SIZE	BM	MOM	AG	ROE	LEV	GO	MAX	TURN	IVOL	EISKEW	EGMB
RET	1.00	-0.01	0.06	0.02	0.01	-0.02	0.01	0.02	-0.01	0.33	0.12	0.00	-0.01	-0.01
BETA		1.00	-0.17	-0.03	-0.03	0.07	-0.11	0.02	0.19	0.24	0.22	0.42	0.21	0.27
SIZE			1.00	-0.31	0.21	0.02	0.18	-0.17	-0.27	-0.38	0.18	-0.66	-0.74	-0.75
BM				1.00	-0.13	-0.11	-0.01	0.47	-0.04	0.11	-0.12	0.08	0.22	0.20
MOM					1.00	-0.04	0.05	-0.09	-0.09	-0.16	0.14	-0.08	-0.26	-0.26
AG						1.00	-0.05	-0.17	0.07	0.02	0.11	0.06	-0.03	-0.02
ROE							1.00	0.07	-0.46	-0.13	-0.02	-0.21	-0.25	-0.26
LEV								1.00	-0.12	0.09	-0.08	0.05	0.14	0.14
GO									1.00	0.24	0.07	0.37	0.36	0.39
MAX										1.00	0.23	0.53	0.40	0.43
TURN											1.00	0.08	-0.14	-0.11
IVOL												1.00	0.71	0.77
EISKEW													1.00	0.97
EGMB														1.00

This table reports summary statistics (Panel A) and correlations (Panel B) for the main variables used in our empirical analyses. RET is the monthly excess return in percentage points; market risk (BETA) is estimated over a 3-year period using the Sharpe-Lintner capital asset pricing model (CAPM) model; SIZE is measured as the natural logarithm of the market value of equity. BM is book-to-market ratio; MOM is momentum, measured as the compound gross return from month t-12 to t-2; AG is asset growth, calculated as the percent change in firm total assets over the previous year; ROE is calculated as the ratio of operating cash flow to shareholders' equity; leverage (LEV) is calculated as the ratio of the book value of debt to the market value of the firm; GO is the growth-option value calculated as per equation (9); MAX is the maximum daily return observed in the previous month; turnover (TURN), calculated as the ratio of trading volume to total shares outstanding; IVOL is the idiosyncratic volatility; EISKEW and EGMB are expected idiosyncratic skewness and expected good minus bad volatility, respectively, estimated from equation (8) over a horizon of 60 months.

Table 2: Cross-Sectional Determinants of Good minus Bad Volatility and Idiosyncratic Skewness

*Panel A.* Good minus Bad Volatility Determinants

	Constant	GMB	GO	ROE	MAX	IVOL	AG	MOM	TURN	SMALL	BIG	INDU	EXCH	$R^2$
	0.056	0.117	0.004	-0.016	0.060	0.014	-0.004	-0.025	-0.003	0.038	-0.030	Yes	Yes	0.28
<i>t</i> -stats	(10.18)	(13.75)	(6.28)	(-9.84)	(5.74)	(13.13)	(-3.90)	(-14.70)	(-4.09)	(20.09)	(-16.48)			

*Panel B.* Idiosyncratic Skewness Determinants

	Constant	GMB	GO	ROE	MAX	IVOL	AG	MOM	TURN	SMALL	BIG	INDU	EXCH	$R^2$
	0.264	0.087	0.042	-0.155	0.613	0.147	-0.028	-0.243	-0.050	0.442	-0.219	Yes	Yes	0.16
<i>t</i> -stats	(4.75)	(11.74)	(4.86)	(-10.37)	(4.33)	(7.77)	(-2.26)	(-10.79)	(-7.16)	(22.83)	(-10.88)			

This table presents the time-series average of the slope coefficients from the monthly cross-sectional regressions of good minus bad volatility (Panel A) or idiosyncratic skewness (Panel B) as in equation (8) from Jan 1978 to December 2020. The corresponding Newey and West (1987) robust *t*-statistics are reported in parentheses. The dependent variable (GMB or ISKEW) is estimated over a period of 5 years ( $T = 60$  months). GO is the growth-option value calculated as per equation (9); ROE proxies for profitability, calculated as the ratio of operating cash flow to shareholders' equity; MAX is the maximum daily return observed in the previous month; Asset growth (AG), calculated as the percent change in firm total assets over the previous year. BM is book-to-market ratio; MOM is momentum, measured as the compound gross return from month  $t-12$  to  $t-2$ ; turnover (TURN), calculated as the ratio of trading volume to total shares outstanding; lagged good minus bad volatility (GMB), calculated in the preceding nonoverlapping 5 years; SMALL and BIG, 2 binary dummies built on the bottom 30% and top 30% of lagged market capitalization; INDU and EXCH are dummies for the Fama and French 30 industries and NASDAQ. The last column reports the average adjusted  $R^2$  values.



Table 3: Comparing Conditional Idiosyncratic Skewness Measures

Panel A. Ex Post Correlations

		$ISKEW_{t-T}$	$GMB_{t-T}$	$EISKEW_t$	$EGMB_t$
$GMB_t$	<i>Pearson</i>	0.252	0.287	0.442	0.464
	<i>Spearman</i>	0.280	0.291	0.437	0.464
$ISKEW_t$	<i>Pearson</i>	0.211	0.222	0.347	0.353
	<i>Spearman</i>	0.255	0.257	0.397	0.415

Panel B. Prediction Errors - Realized Good Minus Bad Volatility

	$ISKEW_{t-T}$	$GMB_{t-T}$	$EISKEW_t$	$EGMB_t$
avg. RMSE	680.90	674.12	600.07	585.46
$ISKEW_{t-T}$		6.78 (0.51)	80.83 (0.94)	95.44 (0.98)
$GMB_{t-T}$			74.05 (0.94)	88.66 (0.98)
$EISKEW_t$				14.61 (0.80)

Panel C. Prediction Errors - Realized Idiosyncratic Skewness

	$ISKEW_{t-T}$	$GMB_{t-T}$	$EISKEW_t$	$EGMB_t$
avg. RMSE	693.96	691.51	620.70	611.51
$ISKEW_{t-T}$		2.45 (0.29)	73.26 (0.96)	82.45 (1.00)
$GMB_{t-T}$			70.81 (0.96)	80.00 (0.99)
$EISKEW_t$				9.19 (0.56)

This table presents the out-of-sample correlation and prediction errors of different measurements. In each month  $t$ , we predict the ex post  $GMB_t$  or  $ISKEW_t$  using daily returns from  $t$  to  $t+T-1$  by 4 methods: lagged realized skewness measures, i.e., i)  $ISKEW_{t-T}$  and ii)  $GMB_{t-T}$  using daily returns from  $t-T$  to  $t-1$ ; iii)  $EISKEW_t$  and iv)  $EGMB_t$  measured as Equation (8) using cross-sectional determinants in  $t-1$ . Panel A reports the average cross-sectional correlations between  $GMB_t$  or  $ISKEW_t$  and the 4 prediction methods. “Pearson” and “Spearman” denote the Pearson correlation and Spearman rank correlation, respectively. Panel B and Panel C report the prediction error for predicting  $GMB_t$  or  $ISKEW_t$ . The first row of Panel B reports the average RMSE over our sample period. The remainder of Panel B present the average differences in RMSE between the model [name in row] and the model [name in column]. The values of the numbers in parentheses indicate the share of time periods for which the difference is significant at 5% (e.g. 0.56 indicates that the difference is significant 56% of the time). We test the significance using the adjusted Diebold-Mariano test (Harvey et al., 1997).

Table 4: Univariate Portfolio Analysis

*Panel A. Univariate Portfolio Sorts based on EGMB*

	1	2	3	4	5	5-1
EGMB	0.04	0.07	0.11	0.15	0.22	0.19
Ret	0.81	0.76	0.55	0.53	-0.09	-0.91
	(4.44)	(3.34)	(2.01)	(1.38)	(-0.20)	(-2.82)
% Mkt Share	65.50	23.30	8.70	2.00	0.40	
FF3	0.18	-0.00	-0.34	-0.57	-1.13	-1.32
	(3.79)	(-0.01)	(-2.74)	(-3.28)	(-4.79)	(-5.01)
<i>q</i> -factor	0.10	0.07	-0.03	0.03	-0.37	-0.47
	(2.33)	(0.99)	(-0.23)	(0.20)	(-1.90)	(-2.34)
FF5	0.08	-0.01	-0.22	-0.22	-0.64	-0.72
	(1.86)	(-0.22)	(-1.73)	(-1.38)	(-3.51)	(-3.68)
FF5+FEISKEW	0.06	0.01	-0.16	-0.11	-0.45	-0.51
	(1.50)	(0.20)	(-1.36)	(-0.74)	(-3.01)	(-3.32)
FF5+FEGMB	0.06	0.02	-0.13	-0.06	-0.38	-0.43
	(1.35)	(0.26)	(-1.13)	(-0.42)	(-2.43)	(-2.66)

*Panel B. Univariate Portfolio Sorts based on EISKEW*

	1	2	3	4	5	5-1
EISKEW	0.06	0.39	0.70	1.14	1.96	1.90
Ret	0.81	0.71	0.62	0.44	0.17	-0.63
	(4.24)	(3.31)	(2.43)	(1.29)	(0.40)	(-2.19)
% Mkt Share	59.20	25.90	11.70	2.60	0.50	
FF3	0.17	-0.03	-0.22	-0.56	-0.82	-0.99
	(3.45)	(-0.47)	(-1.73)	(-3.55)	(-3.49)	(-3.80)
<i>q</i> -factor	0.11	0.05	0.00	-0.03	-0.22	-0.33
	(2.20)	(0.87)	(0.01)	(-0.20)	(-1.05)	(-1.47)
FF5	0.08	-0.03	-0.15	-0.28	-0.47	-0.55
	(1.69)	(-0.48)	(-1.2)	(-1.76)	(-2.25)	(-2.42)
FF5+FEISKEW	0.06	-0.01	-0.10	-0.18	-0.26	-0.31
	(1.33)	(-0.19)	(-0.81)	(-1.25)	(-1.79)	(-2.12)
FF5+FEGMB	0.06	-0.01	-0.07	-0.14	-0.21	-0.27
	(1.29)	(-0.12)	(-0.62)	(-0.98)	(-1.28)	(-1.52)

This table presents the average returns and portfolio characteristics of univariate portfolio analyses. Monthly value-weighted portfolios are formed by sorting all stocks into five quintiles based on the given sort variable, from Jan 1978 to Dec 2020. The first row presents the average EGMB or EISKEW of individual stocks in each quintile. The row labeled “Ret” reports the mean of 1-month ahead excess returns of each portfolio. The row labeled “% Mkt Share” reports the percentage of total market capitalization. The last 5 rows show the risk-adjusted returns (alphas) to 5 different factor models: i) “FF3” is with respect to the market (MKT), size (SMB), book-to-market (HML) factors of Fama and French (1993) ii) Hou et al. (2015) “*q*-factor” model; iii) “FF5” is “FF3” model augmented with investment (CMA), and profitability (RMW) factors of Fama and French (2015); iv) and v) “FF5” model augmented with expected idiosyncratic skewness factor (FEISKEW) or expected good minus bad volatility factor (FEGMB). The column labeled “5-1” reports the difference in returns between portfolio 5 and portfolio 1. The corresponding Newey and West (1987) robust *t*-statistics with 6 lags are reported in parentheses. Panel A displays the results sorted by expected good minus bad volatility EGMB and Panel B by expected idiosyncratic skewness EISKEW.

Table 5: Correlations of Expected Good minus Bad Volatility and Expected Idiosyncratic Skewness Factors with the Five Factors of Fama and French (2015)

	FEGMB	FEISKEW	Mkt-RF	SMB	HML	RMW	CMA
FEGMB	1.00	0.81	-0.26	-0.34	0.19	0.58	0.12
FESKEW	0.81	1.00	-0.23	-0.33	0.15	0.53	0.09
Mkt-RF	-0.26	-0.23	1.00	0.26	-0.22	-0.26	-0.38
SMB	-0.34	-0.33	0.26	1.00	-0.08	-0.40	-0.07
HML	0.19	0.15	-0.22	-0.08	1.00	0.22	0.67
RMW	0.58	0.53	-0.26	-0.40	0.22	1.00	0.11
CMA	0.12	0.09	-0.38	-0.07	0.67	0.11	1.00

The table reports pairwise correlations of Expected Good minus Bad Volatility and Expected Idiosyncratic Skewness Factors with the Five Factors of Fama and French (2015). The expected good minus bad volatility factor (FEGMB) or expected idiosyncratic skewness factor (FEISKEW) is constructed by forming zero cost long-short portfolios associated with the 2 anomalies (EGMB or EISKEW). Mkt-RF is the excess market return, SMB is the size factor, HML is the book-to-market factor, RMW is the profitability factor, and CMA is the investment factor.

Table 6: Bivariate Dependent-Sort Portfolio Analysis

*Panel A.* Sorted by EGMB Controlling for EISKEW

	1	2	3	4	5	5-1	FF3 alpha
1	0.91 (3.98)	0.75 (3.78)	0.81 (4.3)	0.84 (4.18)	0.90 (3.61)	-0.01 (-0.06)	-0.17 (-0.86)
2	0.80 (4.18)	0.72 (3.25)	0.89 (3.44)	0.84 (3.05)	0.53 (1.75)	-0.27 (-1.20)	-0.41 (-1.96)
3	0.71 (3.04)	0.74 (2.60)	0.66 (2.10)	0.53 (1.54)	0.51 (1.21)	-0.20 (-0.61)	-0.58 (-2.16)
4	0.55 (1.85)	0.83 (2.36)	0.69 (1.71)	0.34 (0.82)	-0.09 (-0.21)	-0.65 (-2.19)	-0.91 (-3.78)
5	0.64 (1.66)	0.19 (0.42)	0.42 (0.97)	-0.14 (-0.27)	-0.81 (-1.46)	-1.46 (-3.88)	-1.32 (-3.49)
avg	0.72 (3.09)	0.65 (2.45)	0.69 (2.41)	0.48 (1.51)	0.21 (0.56)	-0.52 (-2.50)	-0.68 (-4.13)

*Panel B.* Sorted by EISKEW Controlling for EGMB

	1	2	3	4	5	5-1	FF3 alpha
1	1.04 (4.21)	0.68 (3.24)	0.75 (3.98)	0.83 (4.46)	0.72 (3.65)	-0.32 (-1.81)	-0.42 (-2.22)
2	0.91 (3.65)	0.78 (3.02)	0.70 (3.04)	0.79 (3.35)	0.69 (2.87)	-0.21 (-1.14)	-0.21 (-1.08)
3	0.51 (1.62)	0.70 (2.33)	0.69 (2.39)	0.64 (2.13)	0.55 (1.86)	0.04 (0.22)	-0.03 (-0.17)
4	0.49 (1.06)	0.52 (1.36)	0.40 (1.11)	0.84 (2.32)	0.65 (1.67)	0.16 (0.50)	0.20 (0.74)
5	-0.23 (-0.46)	0.27 (0.58)	0.05 (0.11)	0.03 (0.06)	-0.42 (-0.94)	-0.20 (-0.66)	-0.01 (-0.02)
avg	0.54 (1.70)	0.59 (2.02)	0.52 (1.85)	0.63 (2.22)	0.44 (1.59)	-0.10 (-0.70)	-0.09 (-0.68)

This table presents the results of bivariate dependent-sort portfolio analyses of the relation between expected good minus bad volatility EGMB and future stock returns after controlling for the expected idiosyncratic skewness EISKEW, and vice versa. In Panel A, for each month, all stocks in the sample are first sorted into 5 quintiles based on an ascending sort of EISKEW. Within each quintile, the stocks are then sorted into 5 quintiles according to EGMB. For each 5×5 grouping, we form a value-weighted portfolio and reports the one-month-ahead excess return. The row “Avg” stands for average EGMB quintile portfolios across the 5 EISKEW portfolios. In Panel B, we reverse the order to first sort on EGMB and then on EISKEW. The column labeled “5-1” reports the difference in the returns between portfolio 5 and portfolio 1. The column labeled “FF3 alpha” reports the average Fama–French 3-factor alphas. The corresponding Newey and West (1987) robust *t*-statistics are reported in parentheses.

Table 7: Cross-Sectional Regressions of Future Returns on EGMB and Control Variables

	BETA	SIZE	BM	EGMB	EISKEW	GO	MOM	ROE	AG	IVOL	COSKEW	MAX
1	0.027 (0.20)	-0.281 (-8.43)	0.129 (2.14)	-6.115 (-4.47)								
2	-0.002 (-0.02)	-0.261 (-7.57)	0.137 (2.24)		-0.585 (-4.72)							
3	-0.021 (-0.15)	-0.147 (-3.73)	0.124 (2.13)			-0.095 (-2.21)						
4	0.034 (0.28)	-0.291 (-8.65)	0.135 (2.33)	-7.486 (-2.04)	0.040 (0.12)							
5	0.046 (0.35)	-0.299 (-9.03)	0.112 (1.97)	-7.140 (-5.59)		-0.048 (-1.24)						
6	0.033 (0.28)	-0.293 (-9.31)	0.137 (2.52)	-6.209 (-4.71)		-0.008 (-0.22)	0.346 (2.15)	0.145 (2.73)	-0.455 (-6.14)			
7	0.098 (0.87)	-0.302 (-9.80)	0.145 (2.66)	-5.787 (-5.01)		0.013 (0.36)	0.352 (2.24)	0.171 (3.23)	-0.466 (-6.34)	-11.444 (-3.97)		
8	0.020 (0.17)	-0.294 (-9.39)	0.139 (2.57)	-5.855 (-4.50)		-0.007 (-0.18)	0.370 (2.41)	0.145 (2.79)	-0.461 (-6.30)		-0.378 (-3.24)	
9	0.110 (0.96)	-0.295 (-9.46)	0.145 (2.64)	-4.859 (-4.02)		0.011 (0.29)	0.317 (1.98)	0.174 (3.25)	-0.462 (-6.21)			-4.394 (-6.50)
10	0.084 (0.74)	-0.274 (-9.12)	0.150 (2.78)	-5.102 (-4.48)		0.008 (0.23)	0.378 (2.52)	0.173 (3.30)	-0.470 (-6.56)	9.322 (2.01)	-0.367 (-3.22)	-6.833 (-6.57)

This table reports the estimated coefficients and Newey and West (1987)  $t$ -statistics (in parentheses) from Fama and MacBeth (1973) cross-sectional regressions. The reported coefficient is the time-series average of month-by-month regression slopes over 516 months (Jan 1978 through December 2020). BETA is the firm's market beta; SIZE is the natural logarithm of the market value of equity; BM is book-to-market ratio; EGMB and EISKEW are expected good minus bad volatility and expected idiosyncratic skewness, respectively, estimated from equation (8) over a horizon of 60 months; GO is the growth-option value calculated as per equation (9); MOM is momentum, measured as the compound gross return from month  $t-12$  to  $t-2$ ; ROE proxies for profitability, calculated as the ratio of operating cash flow to shareholders' equity; AG is asset growth, calculated as the percent change in firm total assets over the previous year; IVOL is the idiosyncratic volatility, estimated with a horizon of 1 month for comparison with Ang et al. (2006); COSKEW is the co-skewness as in Harvey and Siddique (2000); MAX is the maximum daily return observed in the previous month.

Table 8: Predictability of EGMB and EISKEW across Subperiods

	High			Low		
	EGMB	EISKEW	Diff	EGMB	EISKEW	Diff
Ret	0.51 (1.32)	0.17 (0.43)	0.34 (2.50)	1.41 (2.55)	1.24 (2.65)	0.17 (0.96)
FF3	0.86 (3.19)	0.52 (1.75)	0.34 (2.62)	1.72 (4.53)	1.62 (4.58)	0.10 (0.74)
<i>q</i> -factor	0.32 (1.22)	-0.01 (-0.04)	0.33 (2.50)	0.60 (1.99)	0.73 (2.28)	-0.13 (-0.87)
FF5	0.43 (1.85)	0.12 (0.44)	0.31 (2.29)	0.85 (2.96)	0.99 (2.93)	-0.14 (-1.18)

This table reports the excess return and corresponding factor model-adjusted alphas on the long-short portfolios that buy stocks with low skewness (i.e. EGMB or EISKEW) and short stocks with high skewness in subperiods of relative forecast accuracy (RFA) between EISKEW and EGMB. The RFA is measured by the Diebold-Mariano statistics in Section 2.4 which reflects the significance of forecast differences between EGMB and EISKEW for each month  $t$ . The high- and low-periods are classified by the 50th percentiles of RFA. The column labeled “Diff” reports the difference in returns between EGMB and EISKEW long-short portfolios.

Table 9: Controlling for Skewness-Related Factors

*Panel A.* Univariate portfolio sorts based on EGMB

	1	2	3	4	5	5-1
Ret	0.76 (3.83)	0.72 (3.09)	0.53 (1.84)	0.49 (1.29)	-0.19 (-0.43)	-0.95 (-2.86)
FF5	0.07 (1.36)	0.00 (0.06)	-0.14 (-0.97)	-0.11 (-0.63)	-0.55 (-2.77)	-0.61 (-2.83)
FF5+FVIX	0.05 (1.01)	0.02 (0.23)	-0.04 (-0.27)	-0.13 (-0.63)	-0.54 (-2.22)	-0.59 (-2.26)
FF5+PSS	0.03 (0.68)	0.07 (1.18)	-0.03 (-0.2)	0.02 (0.11)	-0.48 (-2.38)	-0.51 (-2.38)

*Panel B.* Univariate portfolio sorts based on EISKEW

	1	2	3	4	5	5-1
Ret	0.75 (3.65)	0.67 (3.0)	0.61 (2.24)	0.40 (1.2)	0.06 (0.14)	-0.69 (-2.31)
FF5	0.07 (1.41)	-0.02 (-0.35)	-0.05 (-0.39)	-0.17 (-0.9)	-0.48 (-2.18)	-0.55 (-2.27)
FF5+FVIX	0.10 (1.96)	-0.07 (-1.08)	-0.01 (-0.07)	-0.17 (-0.8)	-0.47 (-1.79)	-0.57 (-2.03)
FF5+PSS	0.03 (0.61)	0.06 (0.99)	0.07 (0.55)	0.00 (0.0)	-0.39 (-1.76)	-0.41 (-1.77)

This table presents further univariate portfolio analyses after the inclusion of other skewness-related factors. The row labeled “Ret” reports the average 1-month ahead excess returns of each portfolio. The last 3 rows show the risk-adjusted returns (alphas) to 3 different factor models: i) “FF5” is Fama and French (2015) market (MKT), size (SMB), book-to-market (HML), investment (CMA), and profitability (RMW) 5 factor model; ii) “FF5+FVIX” is FF5 model augmented with the aggregate volatility risk factor (FVIX, Barinov (2018)). iii) “FF5+PSS” is FF5 model augmented with the predicted systematic skewness factor (PSS, Langlois (2020)). The column labeled “5-1” reports the difference in returns between portfolio 5 and portfolio 1. The corresponding Newey and West (1987) robust *t*-statistics are reported in parentheses. Panel A displays the results sorted by expected good minus bad volatility EGMB and Panel B by expected idiosyncratic skewness EISKEW.

Table 10: Cross-Sectional Regression of Future Returns on EGJ, EBJ and Control Variables

	BETA	SIZE	BM	EGJ	EBJ	GO	MOM	ROE	AG	IVOL	COSKEW	MAX
1	-0.003 (-0.02)	-0.237 (-6.59)	0.120 (2.03)	-6.041 (-3.65)								
2	0.033 (0.25)	-0.196 (-5.10)	0.163 (2.74)		-9.099 (-3.23)							
3	0.055 (0.42)	-0.276 (-7.95)	0.143 (2.51)	-5.665 (-3.64)	-7.935 (-3.04)							
4	-0.021 (-0.15)	-0.147 (-3.73)	0.124 (2.13)			-0.095 (-2.21)						
5	0.012 (0.09)	-0.249 (-7.05)	0.104 (1.83)	-6.857 (-4.59)		-0.061 (-1.59)						
6	0.051 (0.39)	-0.206 (-5.54)	0.141 (2.52)		-10.428 (-3.99)	-0.078 (-1.98)						
7	0.073 (0.57)	-0.290 (-8.42)	0.126 (2.29)	-6.594 (-4.55)	-8.705 (-3.42)	-0.058 (-1.52)						
8	0.052 (0.45)	-0.287 (-8.75)	0.156 (2.99)	-5.343 (-3.62)	-8.134 (-3.06)	-0.021 (-0.55)	0.346 (2.14)	0.147 (2.83)	-0.478 (-6.32)			
9	0.105 (0.94)	-0.299 (-9.62)	0.166 (3.18)	-4.727 (-3.19)	-7.687 (-3.15)	-0.002 (-0.04)	0.360 (2.28)	0.174 (3.34)	-0.487 (-6.52)	-11.711 (-4.00)		
10	0.040 (0.35)	-0.287 (-8.81)	0.157 (3.05)	-4.881 (-3.35)	-8.082 (-3.06)	-0.020 (-0.53)	0.372 (2.41)	0.149 (2.91)	-0.483 (-6.47)		-0.394 (-3.43)	
11	0.113 (1.00)	-0.295 (-9.23)	0.164 (3.11)	-4.548 (-3.05)	-5.132 (-2.09)	-0.002 (-0.04)	0.325 (2.02)	0.175 (3.35)	-0.476 (-6.31)			-4.459 (-6.68)
12	0.089 (0.79)	-0.270 (-8.91)	0.169 (3.27)	-4.085 (-2.77)	-6.607 (-2.75)	-0.006 (-0.18)	0.390 (2.58)	0.176 (3.41)	-0.487 (-6.69)	9.052 (1.92)	-0.383 (-3.45)	-6.803 (-6.57)

This table reports the estimated coefficients and Newey and West (1987)  $t$ -statistics (in parentheses) from Fama and MacBeth (1973) cross-sectional regressions. EGJ and EBJ are expected good jump and expected bad jump, respectively, estimated from equation (8) over a horizon of 60 months. Other variables are defined the same as Table 7.



Table 11: Cross-Sectional Regression of Future Returns on GO and exposures to macroeconomic uncertainty risks

	BETA	SIZE	BM	GO	$\beta_{bv}$	$\beta_{gv}$	EGMB
1	-0.051 (-0.37)	-0.136 (-3.44)	0.130 (2.20)	-0.094 (-2.16)			
2	-0.053 (-0.39)	-0.148 (-3.73)	0.079 (1.29)	-0.103 (-2.42)	-0.019 (-2.52)		
3	-0.067 (-0.49)	-0.146 (-3.68)	0.079 (1.32)	-0.102 (-2.40)		0.021 (2.33)	
4	-0.055 (-0.41)	-0.159 (-4.03)	0.050 (0.83)	-0.106 (-2.49)	-0.025 (-1.77)	-0.008 (-0.47)	
5	0.002 (0.02)	-0.295 (-8.91)	0.042 (0.70)	-0.063 (-1.66)	-0.029 (-2.13)	-0.013 (-0.79)	-6.445 (-5.02)

Table 12: Investor Sentiment and the Expected Skewness Anomalies

*Panel A.* Predictive Regression for Excess Returns

	1	2	3	4	5	5-1
EISKEW	-0.36 (-1.41)	-0.85 (-2.70)	-1.29 (-3.55)	-1.73 (-3.71)	-2.66 (-4.61)	-2.30 (-5.36)
EGMB	-0.30 (-1.29)	-1.05 (-3.01)	-1.41 (-3.75)	-1.95 (-3.86)	-2.63 (-4.01)	-2.33 (-4.39)

*Panel B.* Predictive Regression for Benchmark-adjusted Returns

	1	2	3	4	5	5-1
EISKEW	0.23 (3.82)	-0.23 (-2.56)	-0.48 (-2.29)	-0.62 (-2.89)	-1.36 (-3.89)	-1.60 (-4.29)
EGMB	0.23 (4.03)	-0.40 (-3.72)	-0.52 (-2.32)	-0.65 (-2.56)	-1.00 (-2.80)	-1.23 (-3.27)

This table reports the results on the relation between the skewness anomalies and market sentiment. In panel A, we estimate the following time-series regression:  $R_{i,t} = a + b * S_{t-1} + u_t$ , where  $R_{i,t}$  is excess return of EGMB (or EISKEW) quintile portfolio  $i$  of month  $t$ .  $S_{t-1}$  is the previous month's level of the investor-sentiment index of Baker and Wurgler (2006). In panel B, we further control for the Fama and French (2015) 5 factors in the regression, i.e.,  $R_{i,t} = a + b * S_{t-1} + c * MKT_t + d * SMB_t + e * HML_t + f * RMW_t + g * CMA_t + u_t$ . We presents the coefficient of the sentiment index ( $b$ ) and its  $t$ -statistics.

## Appendix

### A Brief Theoretical Analysis of the Decomposition

We briefly outline the key theoretical results that allow us to estimate volatilities from positive and negative price increments under the assumption that the underlying continuous-time price process follows a jump diffusion process. We mainly rely on Barndorff-Nielsen et al. (2010).

To set out the notation, let  $p_T$  denote the natural logarithmic price of a security at time  $T$ , which is assumed to follow the generic jump diffusion process,

$$p_T = \int_0^T d + \int_0^T dW + J_T$$

where  $d$  and  $dW$  denote the drift and diffusive volatility processes, respectively,  $W$  is a standard Brownian motion, and  $J$  is a pure jump process. We will denote the natural logarithmic discrete-time return over the time-interval of length  $1/n$  as  $r_{t+i/n} = p_{t+i/n} - p_{t+(i-1)/n}$ .

Andersen et al. (2003) show that the following general results as the sampling frequency  $N$  goes to infinity for realized variance from time  $t-1$  to  $t$

$$RV_t = \sum_{i=1}^n r_{t-1+i/n}^2 \xrightarrow{p} \int_{t-1}^t \frac{2}{3} ds + \sum_{t-1 \leq \cdot \leq t} J^2$$

That is, the realized variance converges to the quadratic variation comprised of the separate components due to “continuous” and “jump” price increments.

Barndorff-Nielsen and Shephard (2002) decompose the total realized variation into separate components associated with the positive and negative high-frequency returns,

$$RV_t^+ = \sum_{i=1}^n r_{t-1+i/n}^2 \mathbf{1}_{\{r_{t-1+i/n} > 0\}}; \quad RV_t^- = \sum_{i=1}^n r_{t-1+i/n}^2 \mathbf{1}_{\{r_{t-1+i/n} < 0\}} \quad (\text{A.1})$$

variation,  $RV_t = RV_t^+ + RV_t^-$ . Moreover, it is possible to show that

$$RV_t^+ \rightarrow \frac{1}{2} \int_{t-1}^t \frac{2}{s} ds + \sum_{t-1 \leq \leq t} J^2 \mathbf{1}_{(J_\tau > 0)}$$

$$RV_t^- \rightarrow \frac{1}{2} \int_{t-1}^t \frac{2}{s} ds + \sum_{t-1 \leq \leq t} J^2 \mathbf{1}_{(J_\tau < 0)}$$

such that the separately defined positive and negative semi-variance measures converge to one-half of the integrated variance plus the sum of squared positive and negative jumps, respectively.

These limiting results imply that the difference between the semi-variance measures removes the variation due to the continuous component and thus only reflects the variation stemming from jumps. The difference between positive and negative semi-variance is also known as the signed jump variation. Feunou et al. (2016) provide theoretical argument that the difference between positive and negative semivariances can be perceived as a measure of realized skewness. It is positive when there are more jumps in the positive return realizations, and it is negative when there are more jumps in the negative return territory.

From Barndorff-Nielsen and Shephard (2006), it can also be shown that the realized skewness measure converges to the jump component raised to the third power.

$$RSK_t \equiv \sum_{i=1}^n r_{t-1+i=n}^3 \xrightarrow{p} \sum_{t-1 \leq \leq t} J^3 \quad \text{as } n \rightarrow \infty \quad (\text{A.2})$$

This result is important in two respects: First, it shows that the realized third moment in the limit separates the jump contribution from the continuous contribution and it just captures the jump part. It does not capture skewness arising from correlation between return and variance innovations (the “leverage” effect). For returns sampled at daily and higher frequencies, such leverage effect are empirically very weak. Second, it shows that the sign of the average jumps matters: firms with more positive jumps on average will have positive skewness.

And bipower variations  $BV_t$ , which unlike realized variance, converges to the continuous part

of the process: the integrated variance

$$BV_t \equiv \sum_{i=2}^n |r_{t-1+i=n}| |r_{t-1+i=n-1=n}| \xrightarrow{p} \int_{t-1}^t \frac{2}{s} ds \quad \text{as } n \rightarrow \infty \quad (\text{A.3})$$

Therefore, using bipower variation make it possible to separate the positive jump component and negative component from the positive realized variance  $RV^+$  and negative realized variance  $RV^-$ .

## B Additional Analysis

### B.1 Sorting on EGMB Controlling for Firm Characteristics

In this section, we conduct dependent-sort analysis to examine the robustness of return pre-

Panel A of Table B.2 presents the value-weighted excess returns of quintile portfolios sorted by lagged realized ISKEW and GMB using data from  $t - T$  to  $t - 1$ . It can be observed that lagged measures do not possess predictive power for future stock returns. For instance, when constructing quintile portfolios based on GMB, the return difference between the highest and lowest groups is only 0.08, with a  $t$ -statistics of 0.46. These results are in line with previous studies.

Panel B of Table B.2 presents the results of sorting on the ex post measures of GMB and ISKEW (using data from period  $t$  to  $t + T - 1$ ). It should be noted that, while these are not tradable strategies, we use them to illustrate the relationship between idiosyncratic skewness and stock returns. As shown in the table, ex post ISKEW and GMB exhibit significant negative associations with stock returns. The return spread between the highest and lowest quintile portfolios based on ex post ISKEW is  $-1.04\%$ , with a highly significant  $t$ -statistics  $-4.89$ . More importantly, consistent with our previous findings, we observe that sorting based on GMB yields a larger difference in portfolio returns:  $-1.37\%$  per month with  $t$ -statistics equal to  $-5.09$ . These results further support that good minus bad volatility could lead to stronger return predictability.

### B.3 Change of Sign of Expected Idiosyncratic Skewness When Controlling for EGMB

To understand how the return predictability of EIKEW changes sign after controlling for EGMB, consider the following data generating process:

$$r_{t+1} = aSK_{t+1} + \epsilon_{t+1} \tag{B.4}$$

$$EISKEW_{t+1} = bSK_t + \epsilon_{t+1} \tag{B.5}$$

$$EGMB_{t+1} = cSK_t + \epsilon_{t+1} \tag{B.6}$$

where  $r_{t+1}$  is the excess stock return for month  $t + 1$ ,  $SK_{t+1}$  is the conditional skewness for returns of month  $t + 1$ ,  $EGMB_{t+1}$  and  $EISKEW_{t+1}$  denotes expected good minus bad volatility and expected idiosyncratic skewness for month  $t + 1$ . Assume that there is a negative relationship between conditional idiosyncratic skewness and stock returns, i.e.,  $a < 0$ , and expected idiosyn-

cratic skewness and expected good minus bad volatility is related to conditional skewness but with measurement errors, thus  $b > 0$  and  $c > 0$ . Now consider the regression of

$$r_{t+1} = \beta_1 EGMB_{t+1} + \beta_2 EISKEW_{t+1} + \epsilon_{t+1} \quad (\text{B.7})$$

The ordinary least squares estimates of  $(\beta_1; \beta_2)$  may be succinctly expressed as:

$$\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \frac{ab}{\sigma_{EGMB}^2 \sigma_{EISKEW}^2 (1 - \rho^2)} \begin{pmatrix} EISKEW(\sigma_{EISKEW}^2 - \frac{c}{b} \sigma_{EGMB}^2) \\ \sigma_{EGMB}^2 (\frac{c}{b} \sigma_{EGMB}^2 - \rho \sigma_{EISKEW}^2) \end{pmatrix}; \quad (\text{B.8})$$

where  $\rho$  is the correlation coefficient between expected idiosyncratic skewness and expected good minus bad volatility. Thus, the  $\beta_2$  estimates would be positive if  $\rho$  and  $\sigma_{EISKEW}^2$  are sufficiently large. To put these numbers to the actual data, equations (B.5) and (B.6) imply that,

$$EGMB = \frac{c}{b} EISKEW + \epsilon_t \quad (\text{B.9})$$

Therefore, regression EGMB on EISKEW results in an estimate of  $\widehat{c/b}$  of 7.96. Moreover,  $\sigma_{EISKEW}^2 - \frac{c}{b} \sigma_{EGMB}^2 = 0.71 - 7.96 \times 0.97 \times 0.07 = 0.17 > 0$  and  $\sigma_{EGMB}^2 (\frac{c}{b} \sigma_{EGMB}^2 - \rho \sigma_{EISKEW}^2) = 7.96 \times 0.07 - 0.97 \times 0.71 = -0.13 < 0$ . Therefore, the change of sign of expected idiosyncratic skewness is consistent with the reasoning that EGMB provides more accurate measure of conditional skewness.

#### B.4 EGMB and the Idiosyncratic Volatility Anomaly

In this section, we investigate whether the return predictability of EGMB helps explain the negative relationship between idiosyncratic volatility (IVOL) and future stock returns (Ang et al., 2006). Previous studies (e.g., Ang et al., 2009; Boyer et al., 2010; Hou and Loh, 2016) have found a positive association between idiosyncratic skewness and idiosyncratic volatility, and argue that this relationship partially explains the IVOL puzzle. Consistent with previous findings, we also find that the coefficients of both variables remain statistically significant when included in the regression (see row 7 in Table 7).

To further investigate the role of skewness in idiosyncratic volatility anomaly, we examine

the idiosyncratic volatility anomaly during our sample period. Table B.3 presents the portfolio characteristics and value-weighted excess returns of IVOL-sorted quintile portfolios. Rows 1 to 3 summarize the average idiosyncratic volatility and two expected skewness measures among each portfolio. We find that IVOL is positively associated with skewness variables, the average EISKEW and EGMB increase monotonically from quintile 1 (Low) to quintile 5 (High). During our sample period, the IVOL anomaly also exists: the return difference between the highest and lowest groups is  $-0.73\%$ , with robust  $t$ -statistics  $-2.39$ . The next two rows show that the results are similar after controlling for the exposures to common asset pricing factors. These results are consistent with previous studies (Ang et al., 2006; Hou and Loh, 2016). In the last two columns, we augment FF5 model with the EISKEW or EGMB factor as we constructed in Section 3.2. We find that skewness factors help to explain the IVOL puzzle. Specifically, after controlling for EISKEW and EGMB factors, the abnormal return of IVOL long-short portfolios decreases to  $-0.44$  ( $t = -2.07$ ) and  $-0.39$  percent ( $t = -1.78$ ), respectively.

We then conduct dependent-sort analysis to examine the return predictability of IVOL after controlling for EGMB, and vice versa. Table Panel A of B.4 presents the results of sorting on IVOL after first sorting on EGMB. We find that, after controlling for EGMB, the relationship between IVOL and returns weakened, particularly in the low EGMB group. For example, the return spread between the highest and the lowest IVOL quintile is  $-0.09\%$  in the lowest EGMB quintile. On average, the return spread between the first and fifth IVOL portfolios is  $0.63\%$  when controlling for EGMB, which is lower than the return of  $0.73\%$  in single sort.

Panel B of Table B.4 repeats the analysis, except that we sort on IVOL first. The results show that controlling for IVOL does reduce the magnitude of the long-short spread of EGMB strategy. The negative relationship between expected good minus bad volatility and future returns can still be observed in all five IVOL sorted quintiles, especially in the high IVOL groups. After controlling for the variation in IVOL, the EGMB sorted long-short portfolio yields  $-0.66\%$  monthly returns on average, with a highly significant  $t$ -statistic of  $-3.35$ , which is also weaker than performance in the single sort analysis. Therefore, our findings are consistent with Boyer et al. (2010), which states that there is some “overlapping” explanatory power in either variable.



Overall, our findings lend support to previous studies that while skewness is helpful in explaining the IVOL puzzle, it does not fully account for the origin of this anomaly. This suggests that there are other factors beyond skewness that can explain this anomaly.

## **B.5 Empirical Methodologies and Return Predictability**

Bali and Cakici (2008) note that the relationship between idiosyncratic volatility and expected returns varies with empirical methodologies, such as weighting schemes, breakpoints utilized to sort stocks, and sample periods. In this section, we investigate whether the relation of EGMB with stock returns is affected by these methodologies.

In Table B.5 columns 1 and 2, we report equal-weighted and value-weighted returns of quintile portfolios sorted by EGMB, respectively. In columns 3 and 4, we form quintile portfolios using different breakpoints. Specifically, we use NYSE breakpoints or generate quintile portfolios with an “equal” market share. In columns 5 to 7, we conduct portfolio sorts after a screen for size, liquidity, and price, respectively. Specifically, to filter the small firms, we exclude all NYSE/AMEX/NASDAQ stocks with market capitalization smaller than the NYSE size decile. Similarly, we exclude illiquidity stocks with Amihud (2002)’s illiquidity belonging to the largest NYSE illiquidity decile. Finally, we exclude stocks whose price is less than \$5. We find that the relationship between EGMB and future stock returns is influenced by different settings. For example, after excluding the small/illiquid stocks, the average return differential between quintile portfolios of the lowest and the highest EGMB is about  $-0.40\%$  per month which is only half of the original premium. The corresponding FF5 factor adjusted returns also decline but are still marginal significant and economically important. Overall, These findings are consistent with previous studies that the idiosyncratic risk-related anomalies are more concentrated in small and illiquid stocks.

## **B.6 Return Predictability of Expected Good and Bad Jumps**

To further investigate the predictive ability of EGJ and EBJ for stock returns, we conduct portfolio sorts and report the value-weighted excess returns of decile portfolios in Panel A of Table

B.6.<sup>17</sup> The return spreads between the highest and lowest groups sorted by EGJ and EBJ are  $-0.77\%$  and  $-0.71\%$ , with  $t$ -statistics of  $-2.10$  and  $-1.71$ , respectively. These results show that regardless of sorting on EGJ or EBJ, the return spread is significantly smaller than that of EGMB-sorted portfolios.

We then employ a double-sort analysis to examine the relationship between expected good jump and expected bad jump. In each month, we sort stocks into three groups by the 30th and 70th percentiles of EBJ (EGJ) and then into deciles by EGJ (EBJ). Panel B of Table B.6 presents the results. The return pattern of sorting on EGJ controlling for EBJ is similar to that of sorting on EBJ controlling for EGJ. Therefore, we do not find much empirical support for the mechanism of downside risk.

Overall, the return patterns suggest that the EGJ and EBJ are both important for the predictability of EGMB. Either component captures partial information of EGMB and hence has weaker return predictability than EGMB.

## B.7 Construction of Macroeconomic Uncertainty

Following Segal et al. (2015), we use the positive and negative realized semivariances of industrial production growth rate  $RV_p$  and  $RV_n$  as respective proxies for realized good and bad uncertainty:

$$RV_{p;t+1} = \sum_{i=1}^N \mathbf{1}_{\{\Delta y_{t+i/N} \geq 0\}} \Delta y_{t+i=N}^2 \quad (\text{B.10})$$

$$RV_{n;t+1} = \sum_{i=1}^N \mathbf{1}_{\{\Delta y_{t+i/N} \leq 0\}} \Delta y_{t+i=N}^2 \quad (\text{B.11})$$

where  $y$  is the monthly industrial production and its demeaned growth rate is  $\Delta y$ ,  $\mathbf{1}_{\{\cdot\}}$  is the indicator function and  $N$  represents the number of observations in a given period  $t$ . We construct realized semi-variance at the annual frequency thus  $N = 12$ .

We then project the logarithm of the future average of  $h$ -periods realized semivariances on the

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<sup>17</sup>We use decile breakpoints because EGJ and EBJ only capture part of variations in EGMB and is subject to more measurement errors. Using quintile breaks yields qualitatively similar results, but with more noise

set of predictors  $X_t$  to obtain ex-ante measures of good and bad uncertainty:

$$\log\left(\frac{1}{h} \sum_{i=1}^h RV_{j;t+i}\right) = const_j + \overset{\circ}{j}X_t + \underset{j;t}{\epsilon}; \quad j = \{p; n\}; \quad (\text{B.12})$$

where the predictors  $X_t$  includes positive and negative realized semivariances  $RV_p$ ,  $RV_n$ , consumption growth  $\Delta c$ , the real-market return  $r_m$ , the market price-dividend ratio  $pd$ , the real risk-free rate  $r_f$ , and the default spread  $def$ . We set the forecast window  $h$  to three years and the definitions of these variables are consistent with Segal et al. (2015). The ex ante good and bad uncertainty  $V_g$  and  $V_b$  are the exponentiated fitted values of the projection:

$$V_{g;t} = \exp(const_p + \overset{\circ}{p}X_t); \quad V_{b;t} = \exp(const_n + \overset{\circ}{n}X_t); \quad (\text{B.13})$$

In addition to measuring the ex ante good and bad uncertainty, we also construct a proxy for the expected consumption growth,  $CG_t$ , as in Segal et al. (2015). Specifically, we project future consumption growth on the same predictors:

$$\frac{1}{h} \sum_{i=1}^h \Delta c_{j;t+i} = const_c + \overset{\circ}{c}X_t + \underset{c;t}{\epsilon}; \quad (\text{B.14})$$

$$CG_t = const_c + \overset{\circ}{c}X_t; \quad (\text{B.15})$$

where  $\Delta c$  corresponds to the growth rate of real per capita expenditures on non-durable goods and services. The consumption data comes from the Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA) tables.

Table B.1: Bivariate Dependent-Sort Portfolio Analysis: Controlling for Firm Characteristics

Control	1	2	3	4	5	Avg
BETA	-0.66 (-2.13)	-0.32 (-1.31)	-0.33 (-1.2)	-0.75 (-2.69)	-1.32 (-3.86)	-0.68 (-3.23)
SIZE	-0.75 (-2.02)	-1.02 (-3.48)	-0.76 (-2.78)	-0.63 (-2.59)	-0.29 (-1.48)	-0.69 (-2.99)
BM	-1.60 (-4.35)	-0.93 (-2.74)	-0.45 (-1.3)	-0.27 (-0.88)	-0.27 (-0.77)	-0.71 (-2.37)
ROE	-1.11 (-2.88)	-0.79 (-2.79)	-0.36 (-1.26)	-0.35 (-1.17)	-0.67 (-1.86)	-0.65 (-2.50)
MOM	-1.28 (-3.25)	-0.55 (-1.64)	-0.62 (-2.12)	-0.17 (-0.54)	-0.54 (-1.81)	-0.63 (-2.43)

This table presents the average excess returns for long-short portfolios that are long stocks in the highest quintile of EGMB and short stocks in the lowest quintile of EGMB within each ‘Control’ variable quintile. The portfolios are formed by sorting all stocks into 5 quintiles based on an ascending sort of the ‘Control’ variable (i.e., BETA, SIZE, BM, ROE, and MOM). Within each quintile, the stocks are then sorted into 5 quintiles according to EGMB. For each 5×5 grouping, we form a value-weighted portfolio and report the one-month-ahead excess return. The column “Avg” stands for average EGMB long-short portfolios across the 5 ‘Control’ variable portfolios.

Table B.2: Return predictability of realized GMB and ISKEW

Table B.3: Univariate Portfolio Analysis: Idiosyncratic Volatility

	1	2	3	4	5	5-1
IVOL	0:01	0:02	0:02	0:03	0:06	0:05
EISKEW	0:43	0:54	0:74	1:03	1:52	1:09
EGMB	0:07	0:09	0:11	0:14	0:19	0:11
Ret	0:78	0:75	0:81	0:52	0:05	-0:73
	(4:67)	(3:31)	(2:81)	(1:43)	(0:13)	(-2:39)
FF3	0:18	-0:04	-0:08	-0:49	-1:01	-1:20
	(4:25)	(-0:58)	(-0:92)	(-3:20)	(-5:10)	(-5:38)
FF5	0:04	-0:05	0:05	-0:20	-0:52	-0:56
	(1:02)	(-0:72)	(0:53)	(-1:60)	(-2:66)	(-2:56)
FF5+FEISKEW	0:03	-0:04	0:07	-0:16	-0:41	-0:44
	(0:71)	(-0:58)	(0:84)	(-1:25)	(-2:16)	(-2:07)
FF5+FEGMB	0:02	-0:03	0:09	-0:13	-0:36	-0:39
	(0:58)	(-0:44)	(0:96)	(-0:98)	(-1:86)	(-1:78)

This table presents the average returns and portfolio characteristics of univariate portfolio analyses. Monthly value-weighted portfolios are formed by sorting all stocks into five quintiles based on IVOL. The first three rows present the average IVOL, EISKEW, and EGMB in each quintile group. The row labeled “Ret” reports the mean of 1-month ahead excess returns of each portfolio. The last 4 rows show the risk-adjusted returns (alphas) to 4 different factor models: i) “FF3” is with respect to the Fama and French (1993) 3-factor model ii) “FF5” is “FF3” model augmented with investment (CMA), and profitability (RMW) factors of Fama and French (2015); iii) and iv) “FF5” model augmented with expected idiosyncratic skewness factor (FEISKEW) or expected good minus bad volatility factor (FEGMB).

Table B.4: Bivariate Dependent-Sort Portfolio Analysis: EGMB and IVOL

*Panel A.* Sorted by IVOL Controlling for EGMB

	1	2	3	4	5	5–1	FF3 alpha
1	0.85 (4.97)	0.86 (4.92)	0.80 (3.56)	0.72 (2.92)	0.76 (2.50)	–0.09 (–0.45)	–0.38 (–2.01)
2	0.84 (4.22)	0.63 (2.37)	0.52 (2.03)	0.84 (2.96)	0.64 (1.87)	–0.20 (–0.80)	–0.54 (–2.41)
3	0.79 (3.36)	0.70 (2.28)	0.57 (1.66)	0.38 (0.99)	0.03 (0.07)	–0.77 (–2.59)	–1.19 (–5.11)
4	0.86 (2.51)	0.65 (1.68)	0.50 (1.14)	–0.06 (–0.13)	0.05 (0.11)	–0.81 (–2.28)	–1.44 (–5.54)
5	0.40 (0.99)	0.25 (0.50)	0.03 (0.05)	–0.73 (–1.38)	–0.90 (–1.66)	–1.30 (–3.30)	–1.53 (–3.56)
avg	0.75 (3.20)	0.62 (2.12)	0.48 (1.48)	0.23 (0.66)	0.11 (0.30)	–0.63 (–2.76)	–1.02 (–5.48)

*Panel B.* Sorted by EGMB Controlling for IVOL

	1	2	3	4	5	5–1	FF3 alpha
1	0.82 (4.75)	0.89 (5.33)	0.80 (4.17)	0.79 (3.65)	0.75 (3.05)	–0.07 (–0.38)	–0.15 (–0.83)
2	0.93 (4.05)	0.66 (2.64)	0.69 (2.69)	0.84 (2.82)	0.67 (2.0)	–0.26 (–1.11)	–0.47 (–2.11)
3	0.99 (3.51)	0.70 (2.37)	0.68 (1.92)	0.39 (1.07)	0.40 (1.0)	–0.59 (–2.22)	–0.64 (–2.59)
4	0.74 (2.14)	0.35 (0.87)	0.25 (0.59)	0.19 (0.43)	–0.03 (–0.05)	–0.77 (–2.26)	–0.82 (–2.45)
5	0.24 (0.63)	0.01 (0.03)	–0.27 (–0.54)	–0.30 (–0.52)	–1.37 (–2.32)	–1.61 (–3.71)	–1.68 (–3.83)
avg	0.74 (2.93)	0.52 (1.83)	0.43 (1.37)	0.38 (1.11)	0.09 (0.23)	–0.66 (–3.35)	–0.75 (–4.13)

This table presents the results of bivariate dependent-sort portfolio analyses of the relation between idiosyncratic volatility (Ang et al., 2006) IVOL and future stock returns after controlling for the expected good minus bad volatility (EGMB), and vice versa. In Panel A, for each month, all stocks in the sample are first sorted into 5 quintiles based on an ascending sort of IVOL. Within each quintile, the stocks are then sorted into 5 quintiles according to EGMB. For each 5×5 grouping, we form a value-weighted portfolio and reports the one-month-ahead excess return. The row “Avg” stands for average EGMB quintile portfolios across the 5 IVOL portfolios. In Panel B, we reverse the order to first sort on IVOL and then on EGMB. The column labeled “5–1” reports the difference in the returns between portfolio 5 and portfolio 1. The column labeled “FF3 alpha” reports the average Fama–French 3-factor alphas. The corresponding Newey and West (1987) robust *t*-statistics are reported in parentheses.

Table B.5: Univariate Portfolio Analysis: Robustness

	Weighting Mechanism		Breakpoints		Firm Screening		
	EW	VW	NYSE	Equal Share	Size	Price	Liquidity
1	1:26 (5:27)	1:17 (6:07)	1:17 (5:98)	1:22 (5:66)	1:18 (5:97)	1:18 (6:06)	1:19 (6:02)
2	1:31 (5:31)	1:11 (4:89)	1:21 (6:01)	1:10 (5:61)	1:17 (5:63)	1:17 (5:44)	1:14 (5:48)
3	1:26 (4:67)	0:91 (3:22)	1:14 (5:06)	1:22 (6:27)	1:10 (4:60)	1:01 (4:19)	1:13 (4:65)
4	1:24 (4:30)	0:89 (2:35)	1:05 (4:15)	0:97 (4:36)	0:95 (3:49)	0:93 (3:23)	0:88 (3:15)
5	0:84 (2:60)	0:26 (0:62)	0:88 (2:56)	0:95 (3:65)	0:77 (2:12)	0:71 (1:98)	0:78 (2:04)
5-1	-0:43 (-2:30)	-0:91 (-2:82)	-0:29 (-1:23)	-0:27 (-1:50)	-0:41 (-1:57)	-0:46 (-1:85)	-0:41 (-1:48)
FF5	-0:12 (-1:00)	-0:72 (-3:85)	-0:36 (-1:92)	-0:24 (-1:33)	-0:36 (-1:99)	-0:37 (-2:11)	-0:35 (-1:82)

This table presents the average excess returns and risk-adjusted alphas of univariate portfolio analyses under different settings. Columns 1 and 2 report equal-weighted and value-weighted returns of quintile portfolios sorted by EGMB, respectively. Column 3 and 4 use NYSE breakpoints or generate quintile portfolios with an “equal” market share. Column 5 to 7 conduct portfolio sorts after a screen for size, liquidity, and price, respectively. The last row “FF5” presents the risk-adjusted returns (alphas) of difference in returns between portfolio 5 and portfolio 1 to Fama and French (2015) 5-factor model.



Table B.6: Return predictability of Expected Good and Bad Jumps

*Panel A. Single Portfolio Sort*

	1	2	3	4	5	6	7	8	9	10	10-1
<i>EGJ</i>	0.73 (3.70)	0.69 (3.78)	0.80 (3.56)	0.81 (3.37)	0.93 (3.40)	0.86 (2.66)	0.66 (1.91)	0.69 (1.91)	0.42 (0.97)	-0.05 (-0.10)	-0.77 (-2.10)
<i>EBJ</i>	0.83 (3.74)	0.80 (3.95)	0.79 (3.57)	0.60 (2.76)	0.62 (2.51)	0.70 (2.86)	0.46 (1.63)	0.30 (0.92)	0.15 (0.34)	0.12 (0.23)	-0.71 (-1.71)

*Panel B. Bivariate Dependent Sort*

EBJ	EGJ			EGJ	EBJ		
	1	10	10-1		1	10	10-1
1	1.17 (3.59)	0.77 (3.55)	-0.40 (-1.53)	1	1.13 (4.03)	0.70 (1.70)	-0.43 (-1.34)
2	0.98 (3.26)	0.59 (2.36)	-0.39 (-1.38)	2	0.88 (3.51)	0.21 (0.49)	-0.67 (-2.01)
3	0.66 (1.89)	-0.32 (-0.64)	-0.98 (-2.50)	3	0.68 (3.00)	-0.24 (-0.45)	-0.92 (-1.94)

This table investigates the return predictability of EGJ and EBJ, along with their relationship. Panel A reports the value-weighted excess returns for each decile portfolio, as well as the 10 – 1 return spreads sorted on EGJ or EBJ, with Newey and West (1987) adjusted *t*-statistics in parentheses. Panel B presents the results of 3 × 10 double-sort analysis where we sort on EBJ 30th and 70th percentiles and then on EGJ deciles (left panel), and vice versa (right panel).